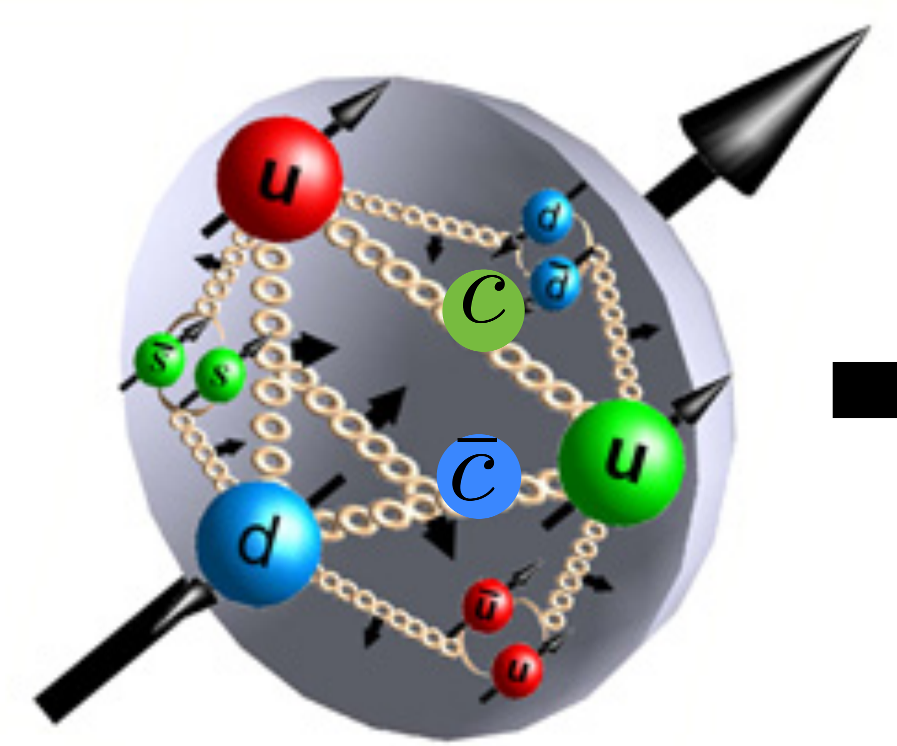
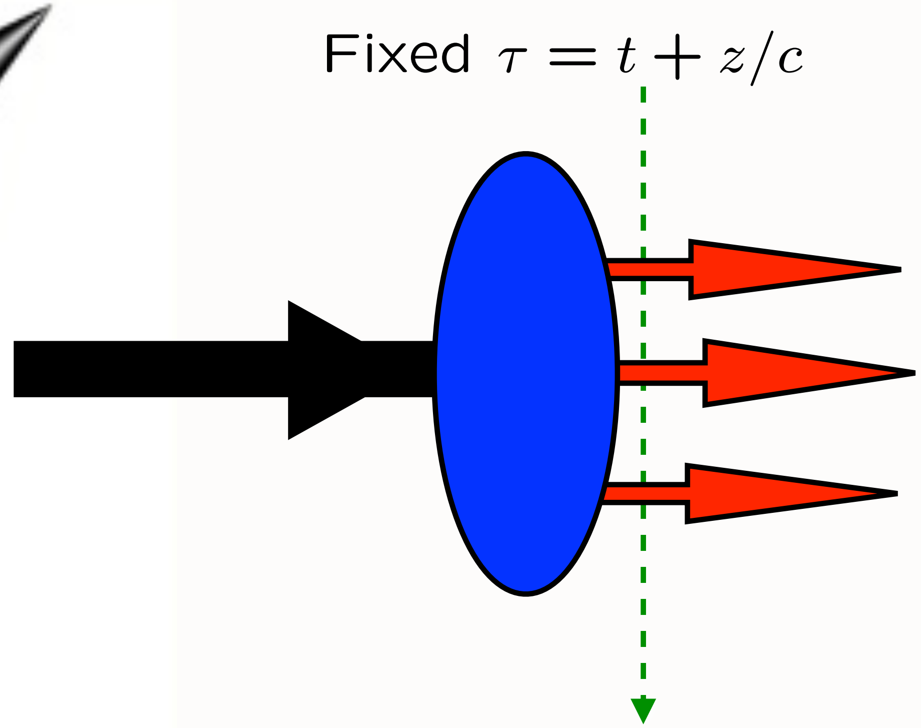
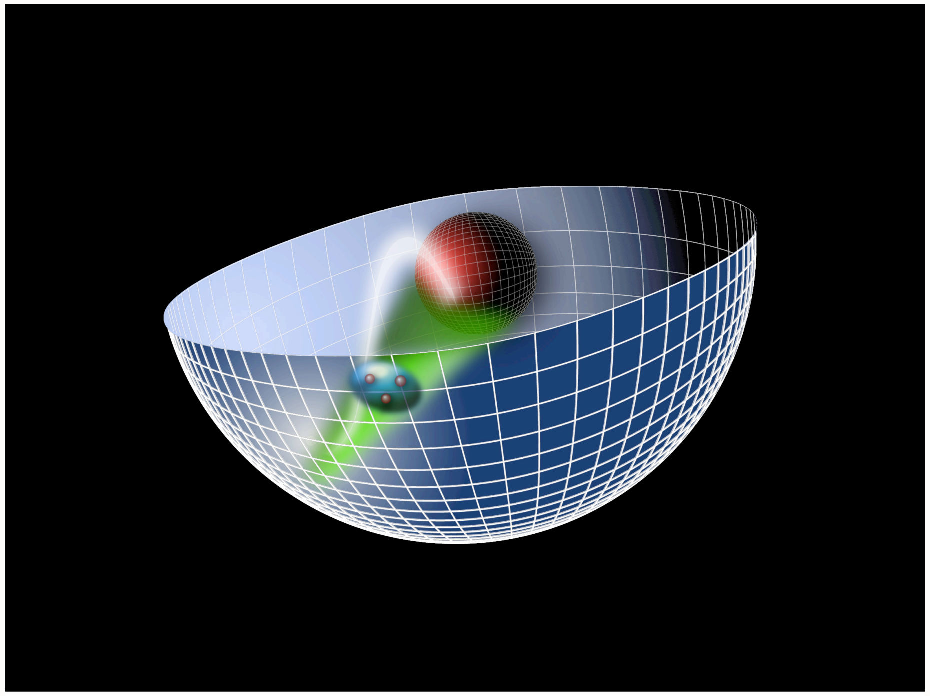
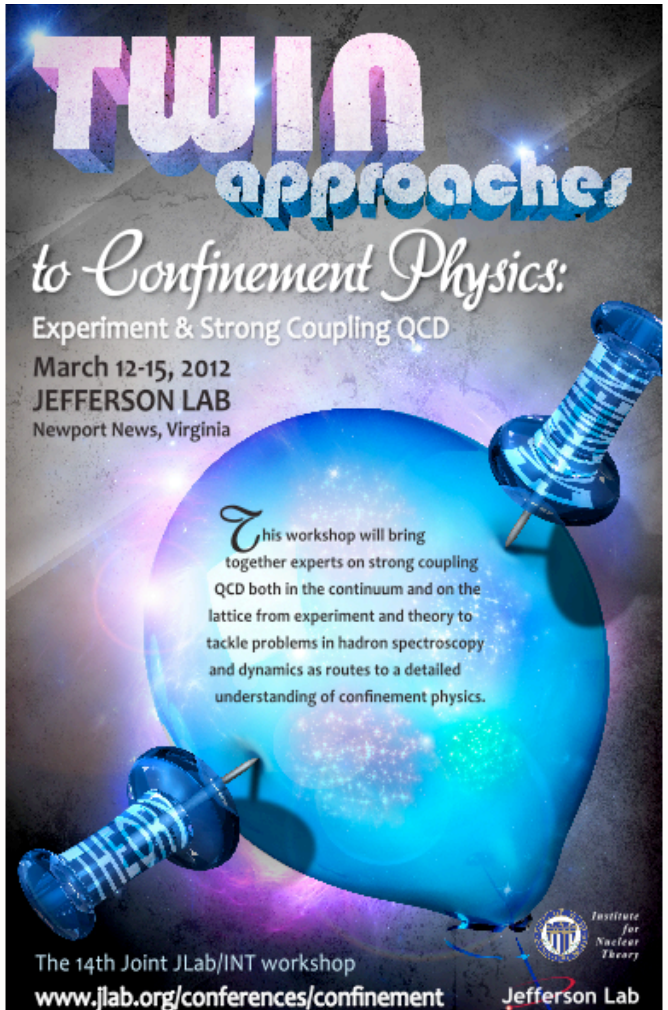
AdS/QCD and Light-Front Holography:

A Novel Approach to Confinement and Non-Perturbative QCD









Workshop on Confinement Physics

March 12-15, 2012

Thomas Jefferson National Accelerator Facility

Newport News, VA

Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

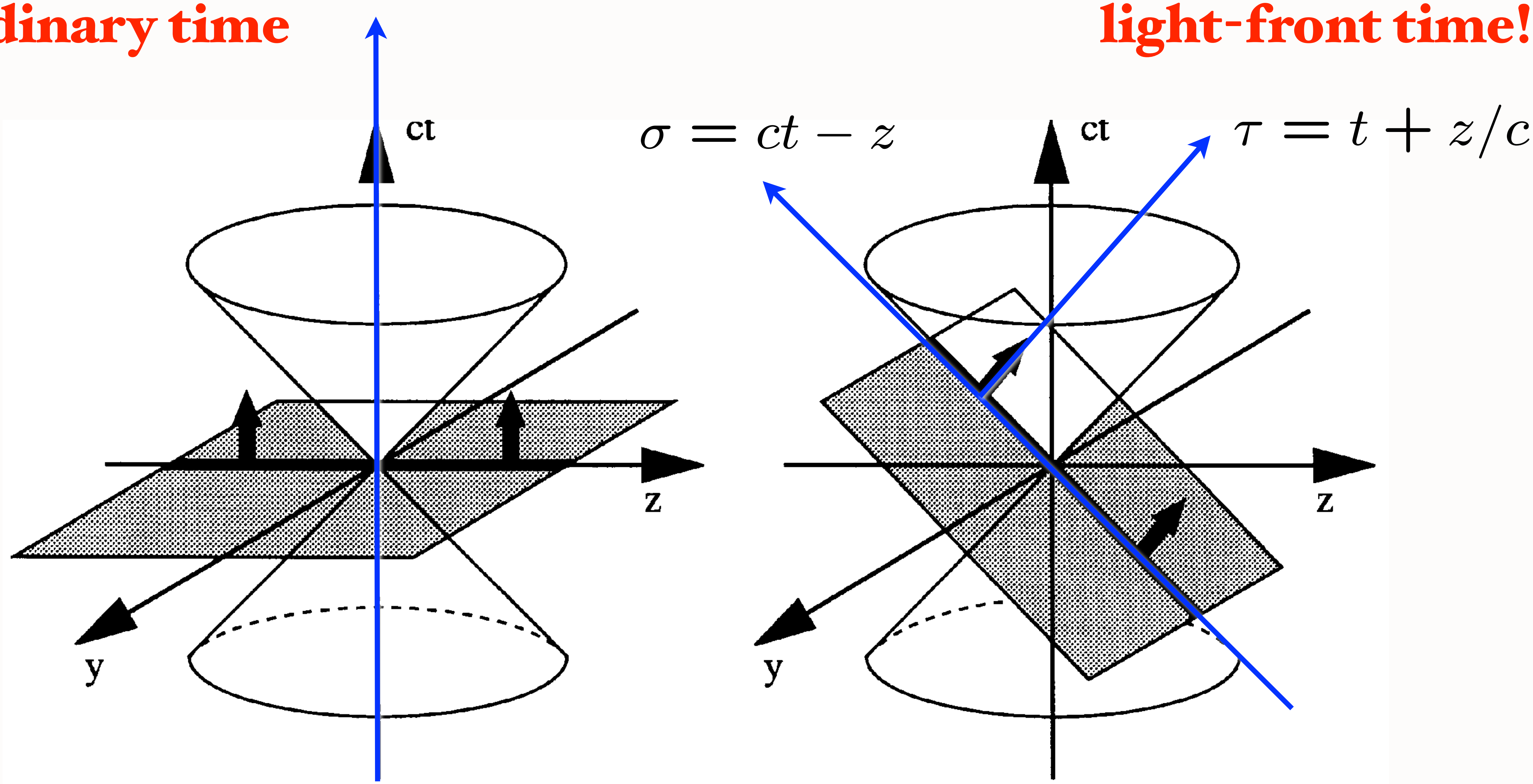
Goal: An analytic first approximation to QCD

- **As Simple as Schrödinger Theory in Atomic Physics**
- **Relativistic, Frame-Independent, Color-Confining**
- **QCD Coupling at all scales**
- **Hadron Spectroscopy**
- **Light-Front Wavefunctions**
- **Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates**
- **Systematically improvable**

Dirac's Amazing Idea: The Front Form

Evolve in ordinary time

Evolve in light-front time!



Instant Form **Front Form**

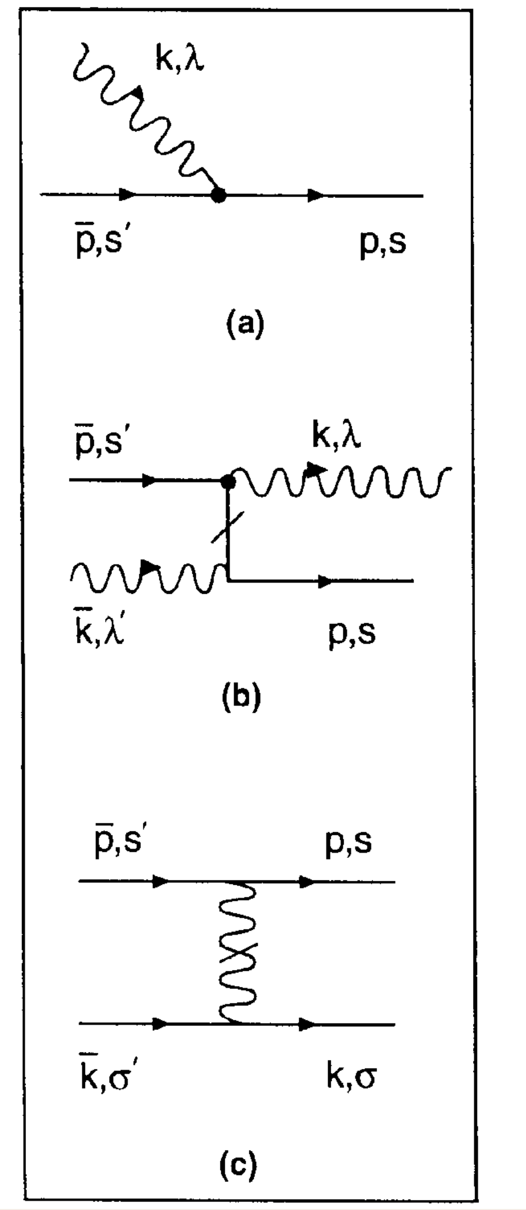
Light-Front QCD

Exact formulation of nonperturbative QCD

Physical gauge: $A^+ = 0$

$$L^{QCD} \to H_{LF}^{QCD}$$

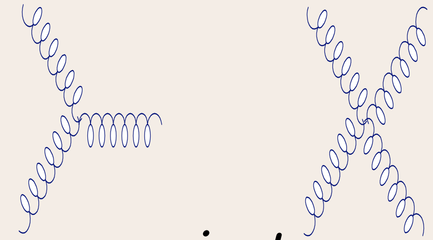
$$H_{LF}^{QCD} = \sum_i [\frac{m^2 + k_\perp^2}{x}]_i + H_{LF}^{int}$$



H_{LF}^{int}: Matrix in Fock Space

$$H_{LF}^{QCD} |\Psi_h> = \mathcal{M}_h^2 |\Psi_h>$$

Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions



H_{LF}^{int}

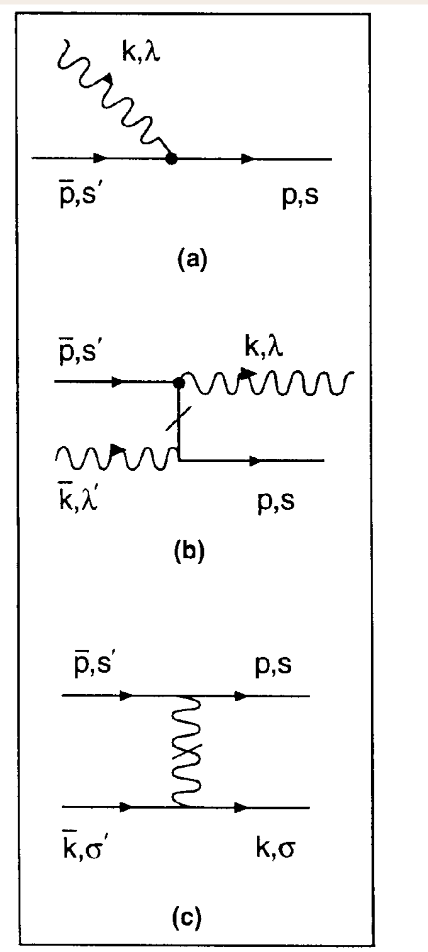
Light-Front QCD

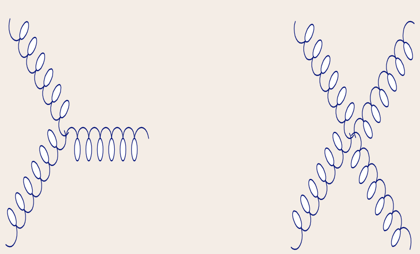
Heisenberg Matrix Formulation

$$\boxed{H^{QCD}_{LC}\,|\Psi_h\rangle = \mathcal{M}^2_h\,|\Psi_h\rangle}$$

DLCQ

Discretized Light-Cone Quantization





n	Sector	1 $q\bar{q}$	2 gg	3 $q\bar{q}\,g$	4 $q\bar{q}\,q\bar{q}$	5 $gg\,g$	6 $q\bar{q}\,gg$	7 $q\bar{q}\,q\bar{q}\,g$	8 $q\bar{q}\,q\bar{q}\,q\bar{q}$	9 $gg\,gg$	10 $q\bar{q}\,gg\,g$	11 $q\bar{q}\,q\bar{q}\,gg$	12 $q\bar{q}\,q\bar{q}\,q\bar{q}\,g$	13 $q\bar{q}\,q\bar{q}\,q\bar{q}\,q\bar{q}$
1	$q\bar{q}$					•		•	•	•	•	•	•	•
2	gg				•			•	•		•	•	•	•
3	$q\bar{q}\,g$								•	•		•	•	•
4	$q\bar{q}\,q\bar{q}$		•			•				•	•		•	•
5	$gg\,g$	•			•			•	•			•	•	•
6	$q\bar{q}\,gg$								•				•	•
7	$q\bar{q}\,q\bar{q}\,g$	•	•			•				•				•
8	$q\bar{q}\,q\bar{q}\,q\bar{q}$	•	•	•		•	•			•	•			
9	$gg\,gg$	•		•	•			•	•			•	•	•
10	$q\bar{q}\,gg\,g$	•	•		•				•				•	•
11	$q\bar{q}\,q\bar{q}\,gg$	•	•	•		•				•				•
12	$q\bar{q}\,q\bar{q}\,q\bar{q}\,g$	•	•	•	•	•	•			•	•			
13	$q\bar{q}\,q\bar{q}\,q\bar{q}\,q\bar{q}$	•	•	•	•	•	•	•		•	•	•		

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

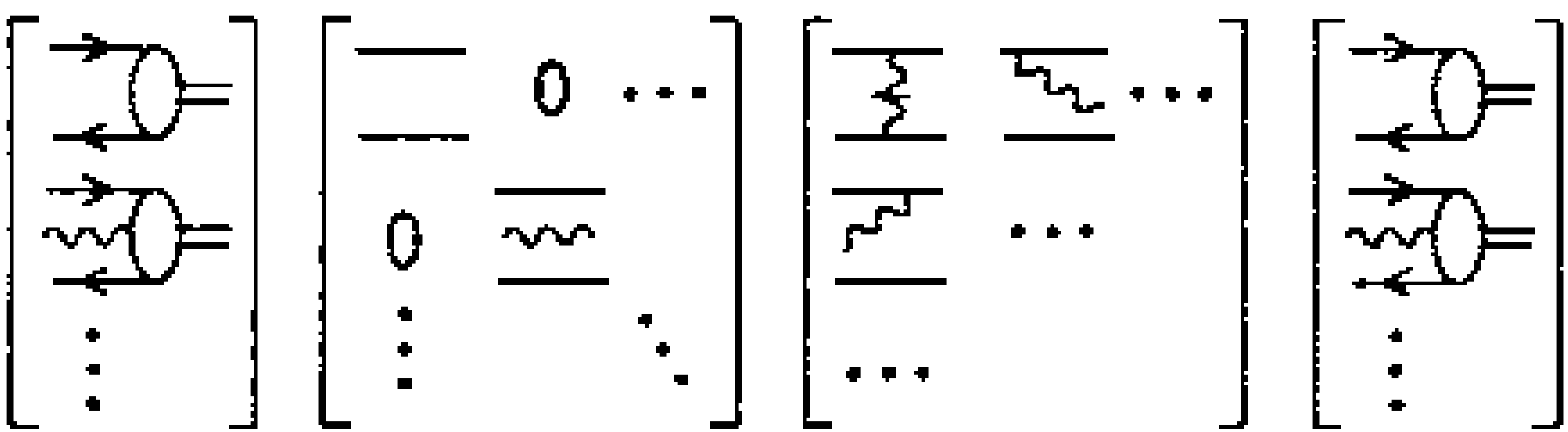
Pauli, Hornbostel & sjb

e.g. solve QCD(1+1): arbitrary color, flavor, quark mass

LIGHT-FRONT SCHRODINGER EQUATION

Direct connection to QCD Lagrangian

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i}\right)\begin{bmatrix}\psi_{q\bar{q}/\pi}\\ \psi_{q\bar{q}g/\pi}\\ \vdots\end{bmatrix} = \begin{bmatrix} \langle q\bar{q}|\,V\,|q\bar{q}\rangle & \langle q\bar{q}|\,V\,|q\bar{q}g\rangle & \cdots \\ \langle q\bar{q}g|\,V\,|q\bar{q}\rangle & \langle q\bar{q}g|\,V\,|q\bar{q}g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}\begin{bmatrix}\psi_{q\bar{q}/\pi}\\ \psi_{q\bar{q}g/\pi}\\ \vdots\end{bmatrix}$$



$A^+ = 0$

G.P. Lepage, sjb

Eigensolutions of the LF Hamiltonian:

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; \vec{k}_{\perp_i}, \lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

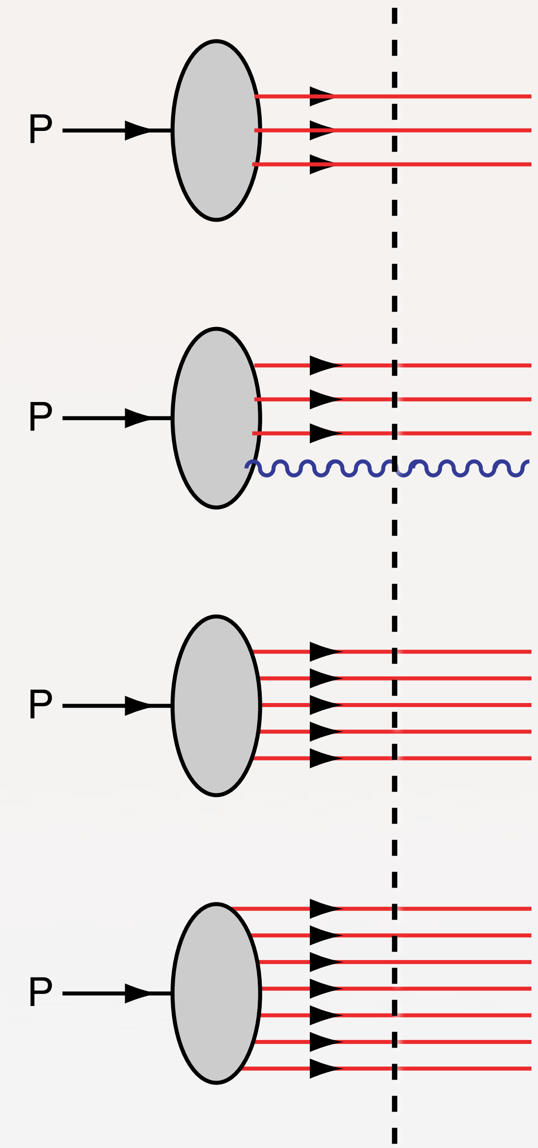
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Intrinsic heavy quarks

s(x), c(x), b(x) at high x !

$\bar{s}(x) \neq s(x)$

$\bar{u}(x) \neq \bar{d}(x)$

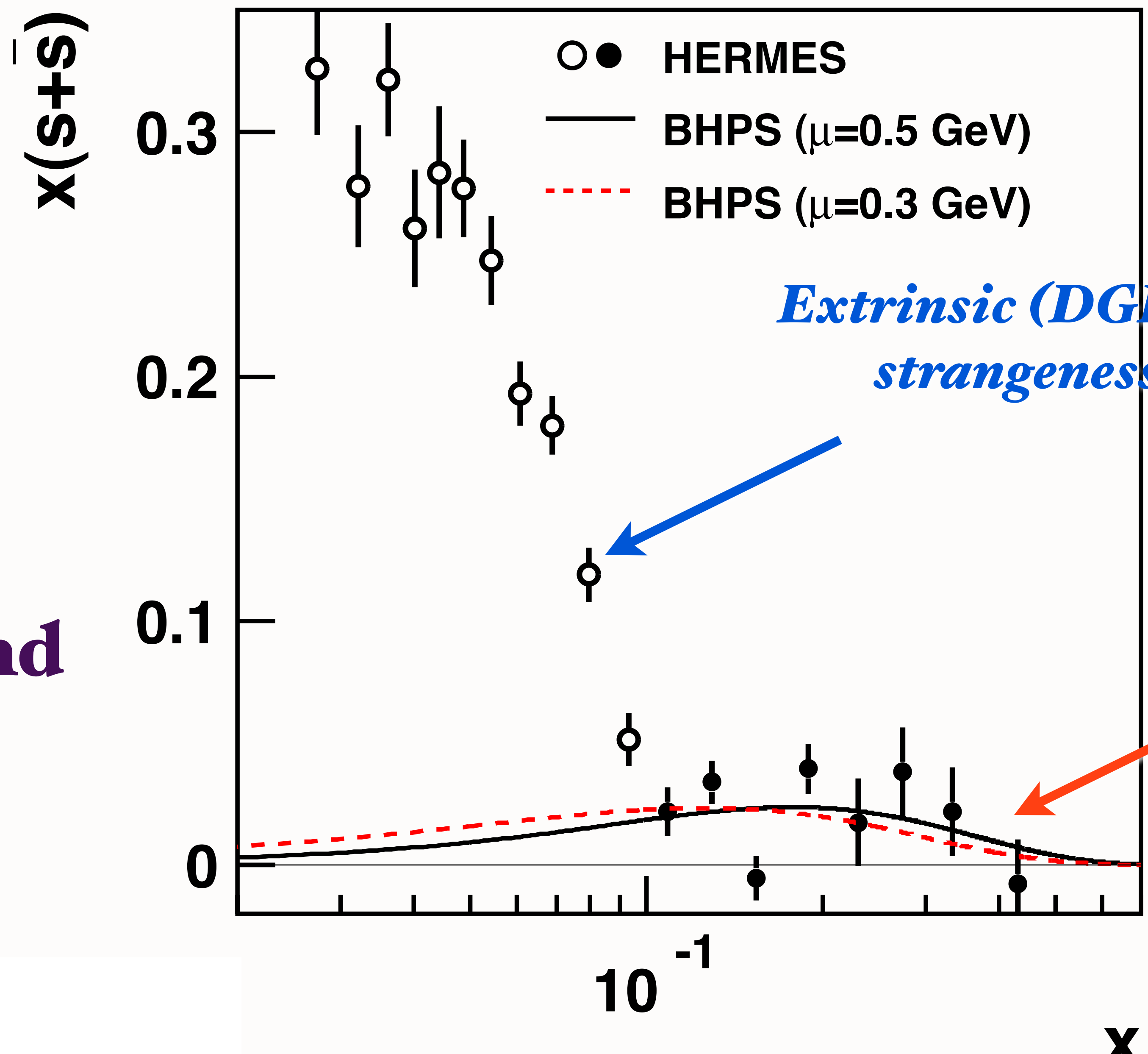
Fixed LF time

Coupled. infinite set

Nuclei: Hidden Color

Mueller: gluonic Fock states >> BFKL

HERMES: Two components to $s(x,Q^2)$!



Extrinsic (DGLAP) strangeness!

Intrinsic strangeness!

Consistent with intrinsic charm data

W. C. Chang and J.-C. Peng

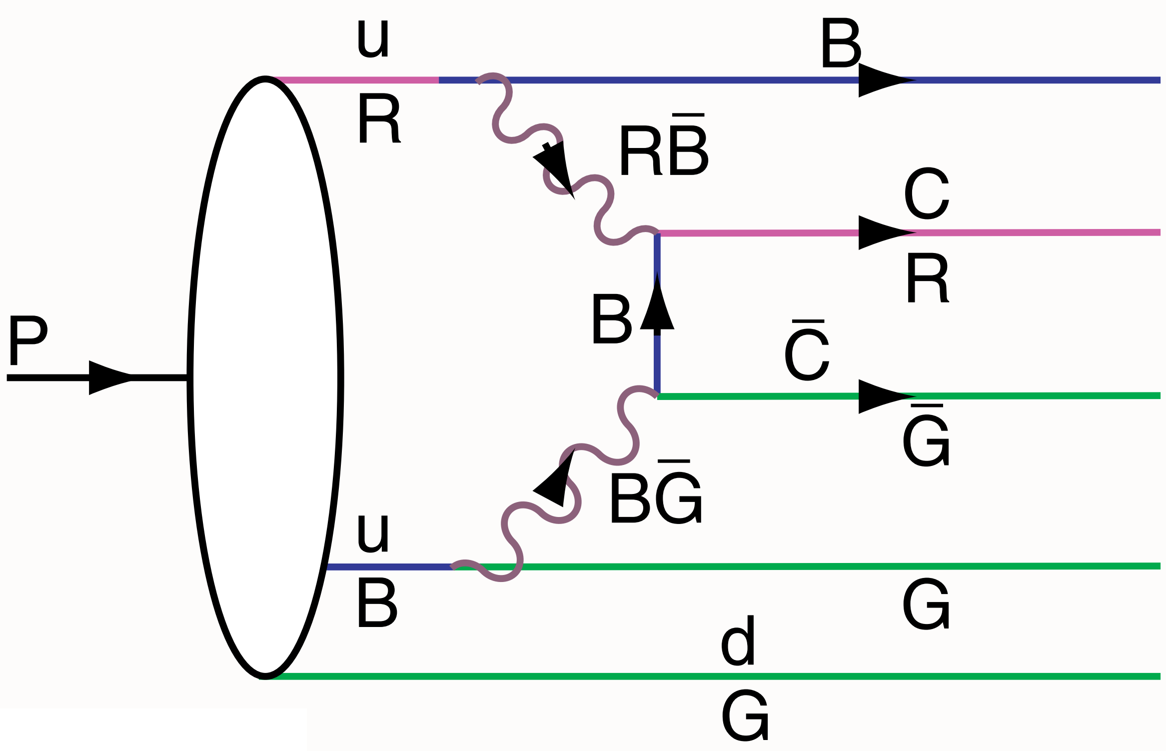
arXiv:1105.2381

Comparison of the HERMES $x(s(x)+\bar{s}(x))$ data with the calculations based on the BHPS model. The solid and dashed curves are obtained by evolving the BHPS result to $Q^2 = 2.5$ GeV^2 using $\mu = 0.5$ GeV and $\mu = 0.3$ GeV, respectively. The normalizations of the calculations are adjusted to fit the data at $x > 0.1$ with statistical errors only, denoted by solid circles.

QCD: $\frac{1}{M_Q^2}$ scaling

$$s(x,Q^2) = s(x,Q^2)_{\text{extrinsic}} + s(x,Q^2)_{\text{intrinsic}}$$

BHPS: Hoyer, Peterson, Sakai, sjb



$|uudc\bar{c}>$ Fluctuation in Proton

QCD: Probability $\frac{\sim\Lambda^2_{QCD}}{M^2_Q}$

$|e^+e^-\ell^+\ell^->$ Fluctuation in Positronium

QED: Probability $\frac{\sim(m_e\alpha)^4}{M^4_\ell}$

OPE derivation - M.Polyakov et al.

$< p|\frac{G^3_{\mu\nu}}{m^2_Q}|p >$ vs. $< p|\frac{F^4_{\mu\nu}}{m^4_\ell}|p >$

$c\bar{c}$ in Color Octet

$$\hat{x}_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

Distribution peaks at equal rapidity (velocity)

Therefore heavy particles carry the largest momentum fractions

$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$

High x charm! *JLab: Charm at Threshold*

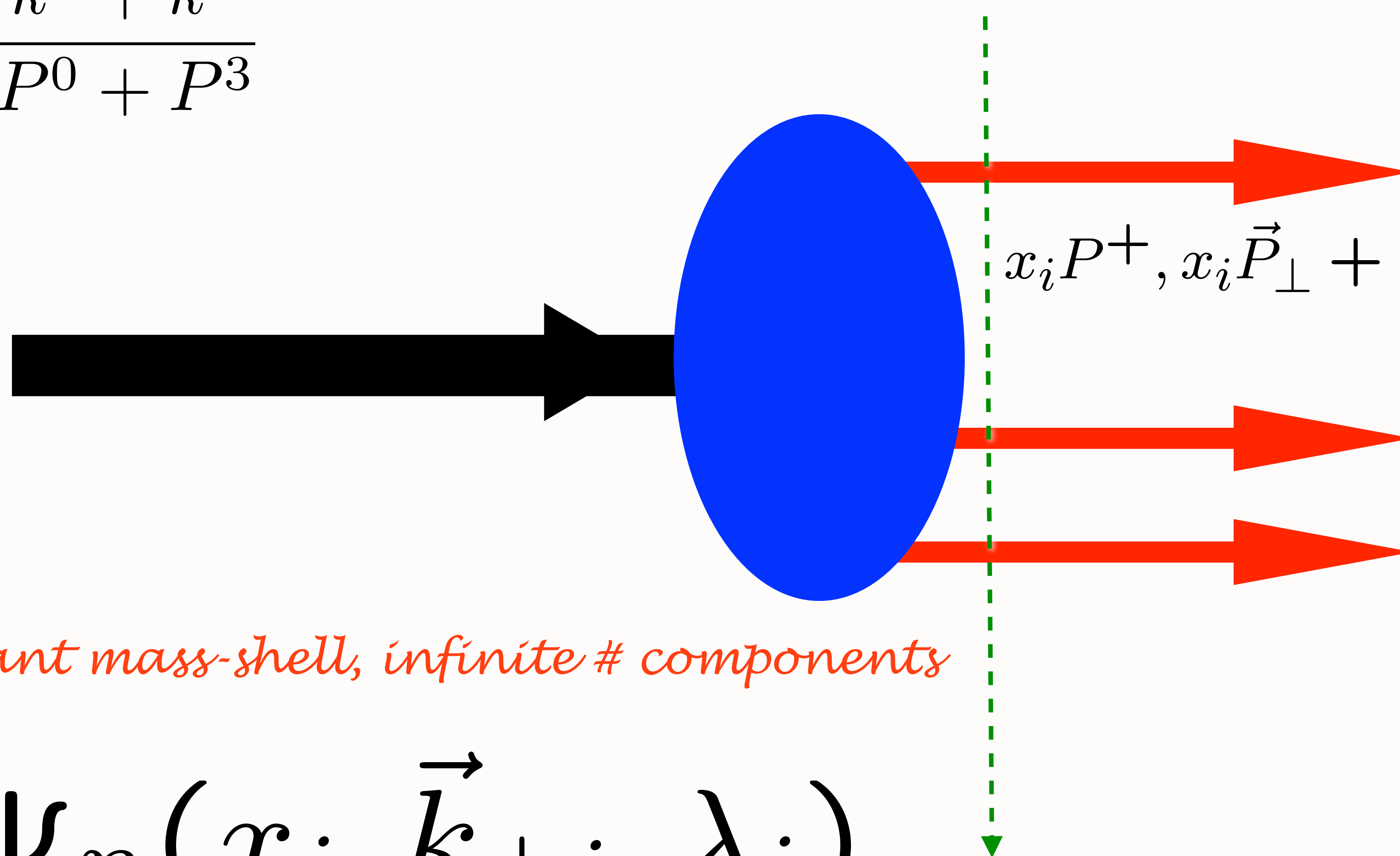
Action Principle: Minimum KE, maximal potential

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed $\tau = t + z/c$

$P^+, \vec{P}_\perp$



$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

LFWFs: off invariant mass-shell, infinite # components

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

Conserved in each LF Fock state

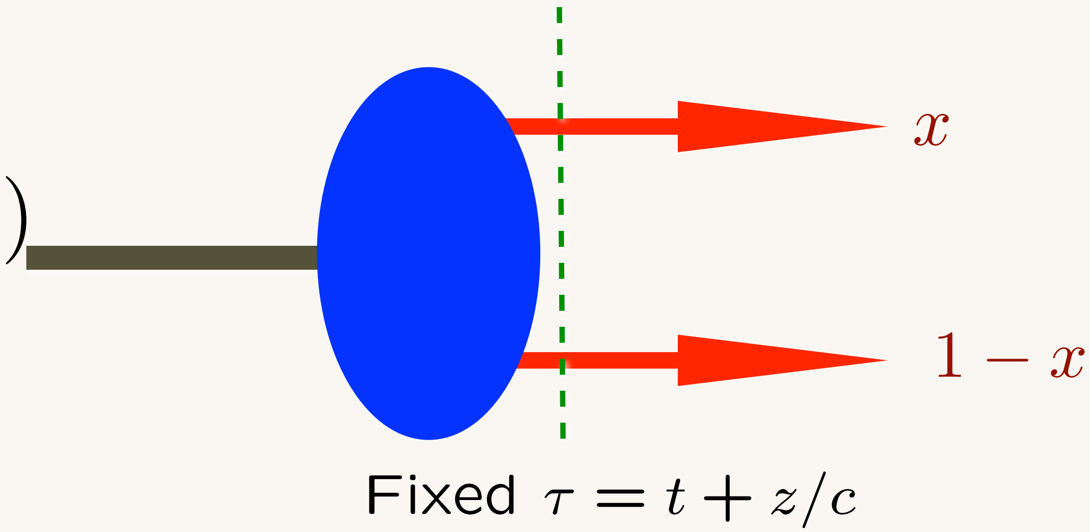
$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Nonzero Anomalous Moment -->Nonzero orbital angular momentum

Hadron Distribution Amplitudes

$$\phi_M(x,Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x,\vec{k}_\perp)$$



$k_\perp^2 < Q^2$

$$\sum_i x_i = 1$$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

Lepage, sjb

- Evolution Equations from PQCD, OPE

Lepage, sjb

Efremov, Radyushkin

Sachrajda, Frishman Lepage, sjb

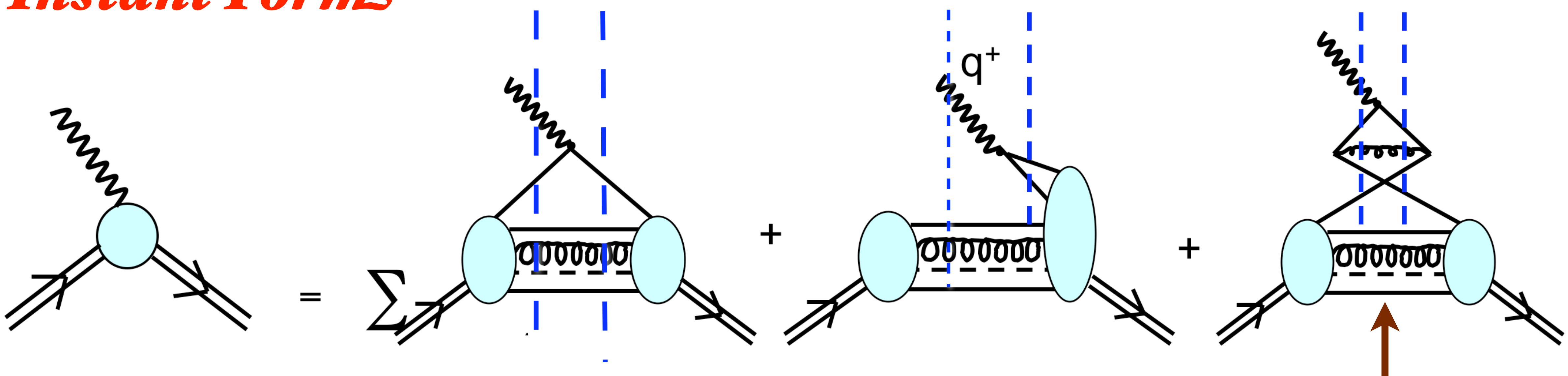
- Conformal Invariance

Braun, Gardi

- Compute from valence light-front wavefunction in light-cone gauge

Calculation of Form Factors in Equal-Time Theory

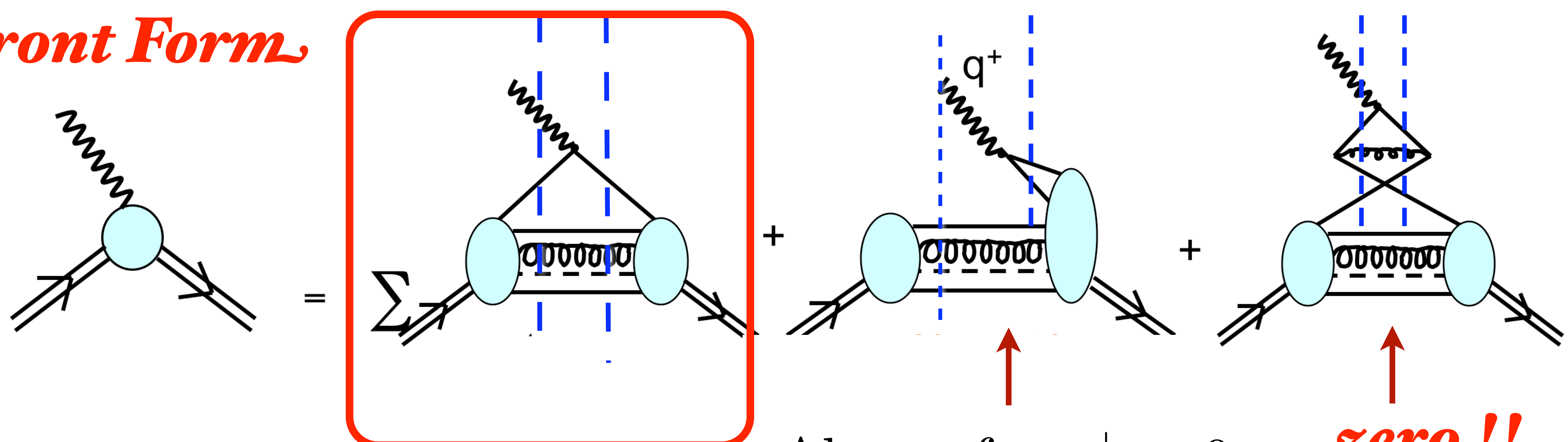
Instant Form



Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

Front Form



Complete Answer

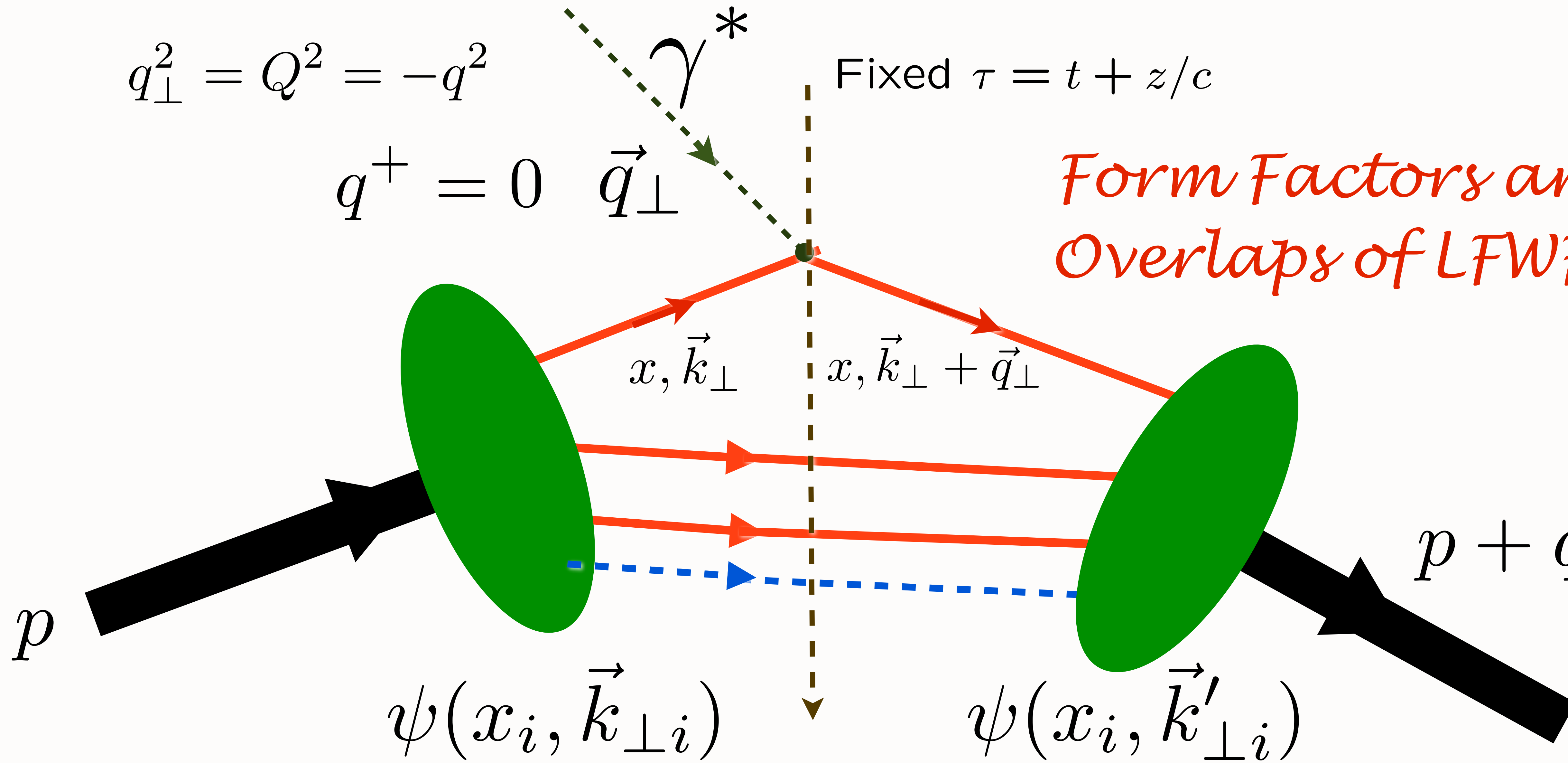
Absent for $q^+ = 0$

zero !!

No vacuum graphs

$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Interaction picture



Form Factors are Overlaps of LFWFs

struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell, Yan; West

Exact LF Formula for Pauli Form Factor

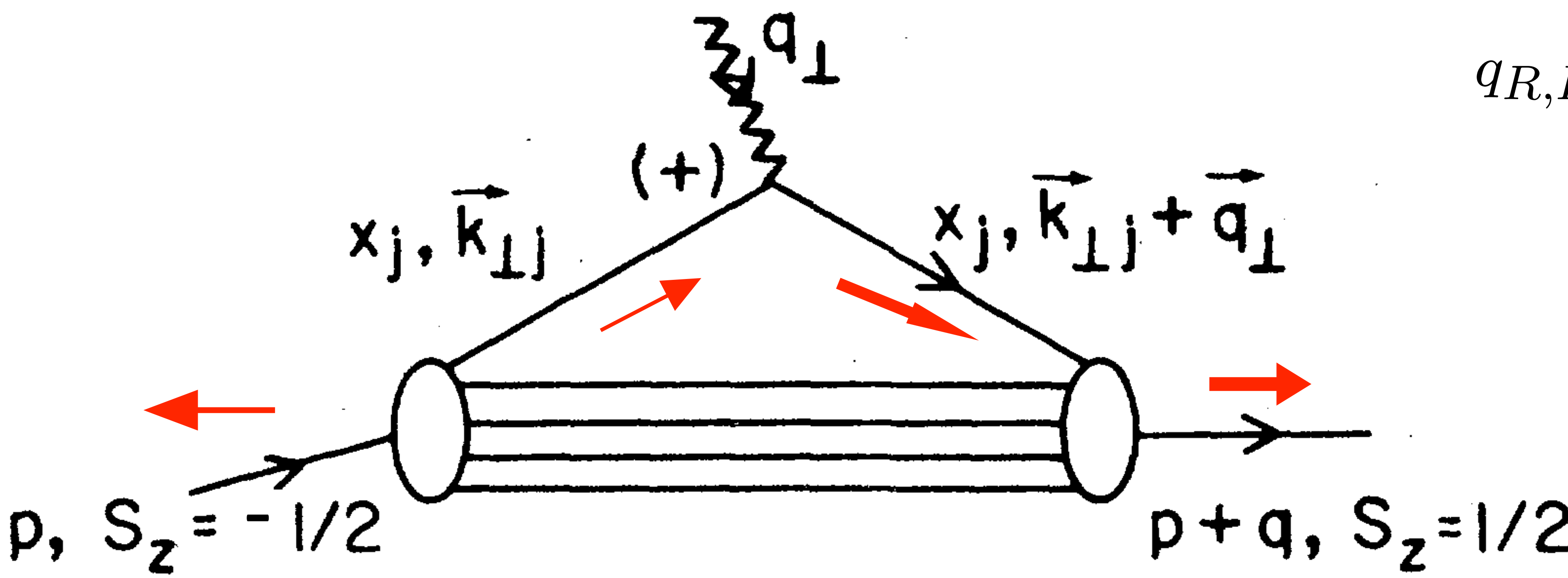
$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\Big[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \Big]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$

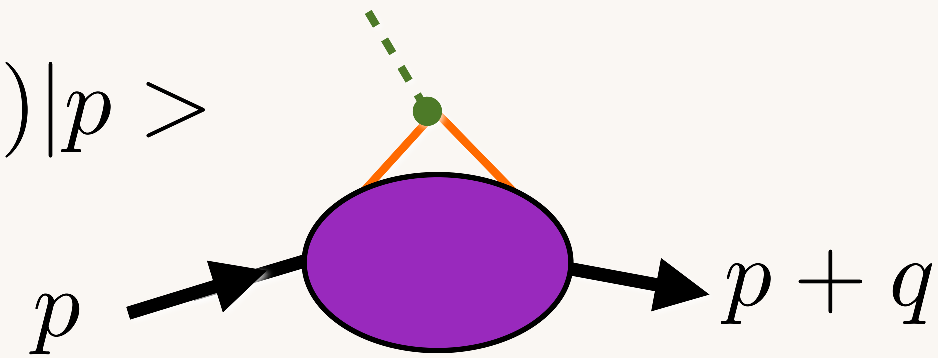


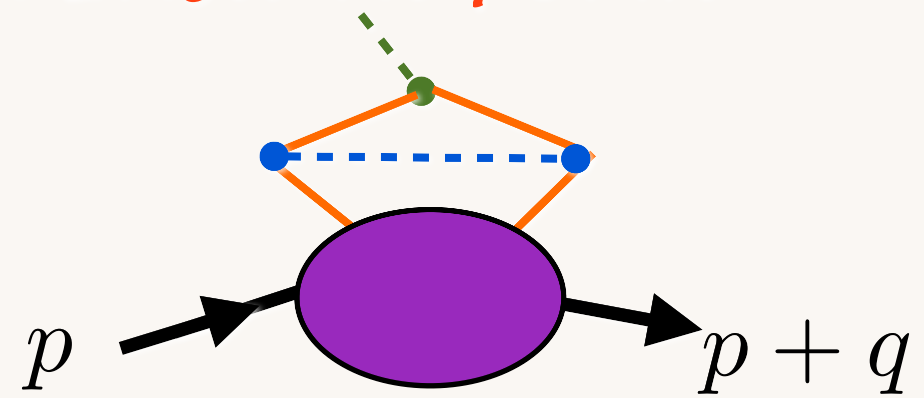
$\boxed{\text{Must have } \Delta\ell_z = \pm 1 \text{ to have nonzero } F_2(q^2)}$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

Calculation of proton form factor in Instant Form

$$< p+q|J^\mu(0)|p >$$





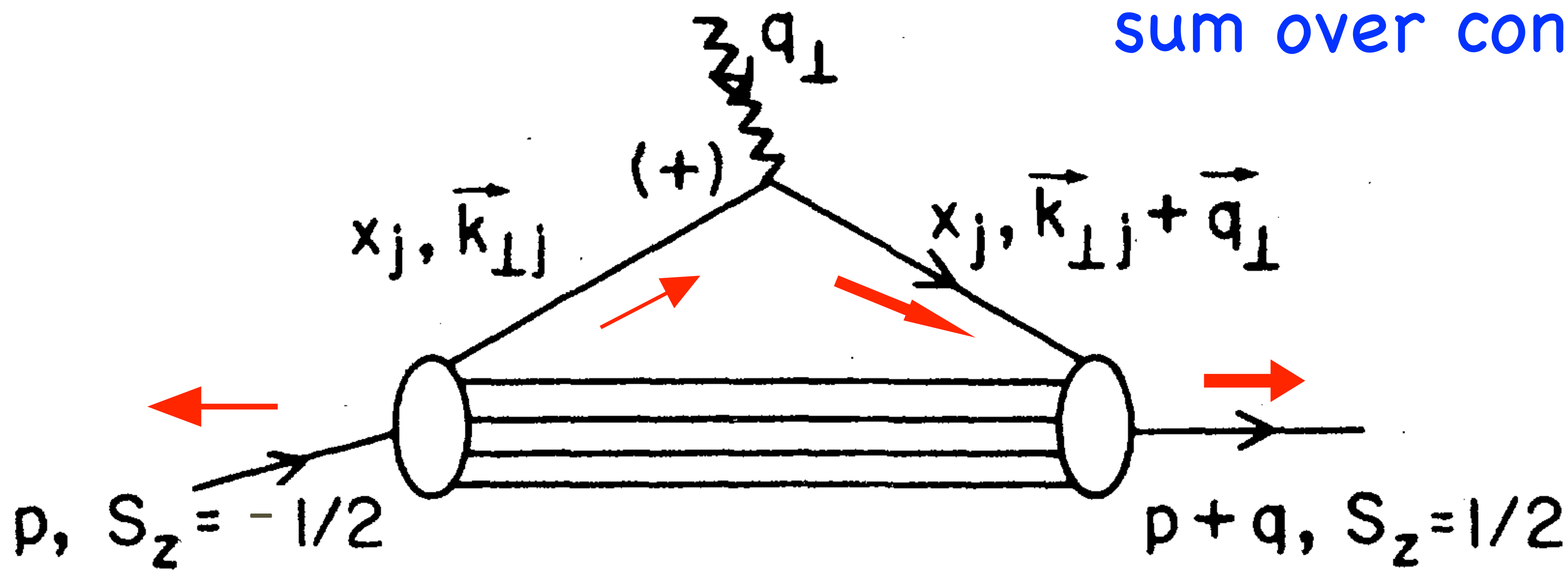
- **Need to boost proton instant form wavefunction from p to p+q: Extremely complicated dynamical problem; particle number changes**
- **Need to couple to all currents arising from vacuum!**
- **Wavefunctions alone do not determine hadronic properties!**
- **Each time-ordered contribution is frame-dependent**
- **None of these problems occur in the front form!**

Anomalous gravitomagnetic moment B(0)

Terayev, Okun, et al: *B(0) Must vanish because of Equivalence Theorem*

graviton

sum over constituents



Hwang, Schmidt, Ma, sjb

$\boxed{B(0) = 0}$

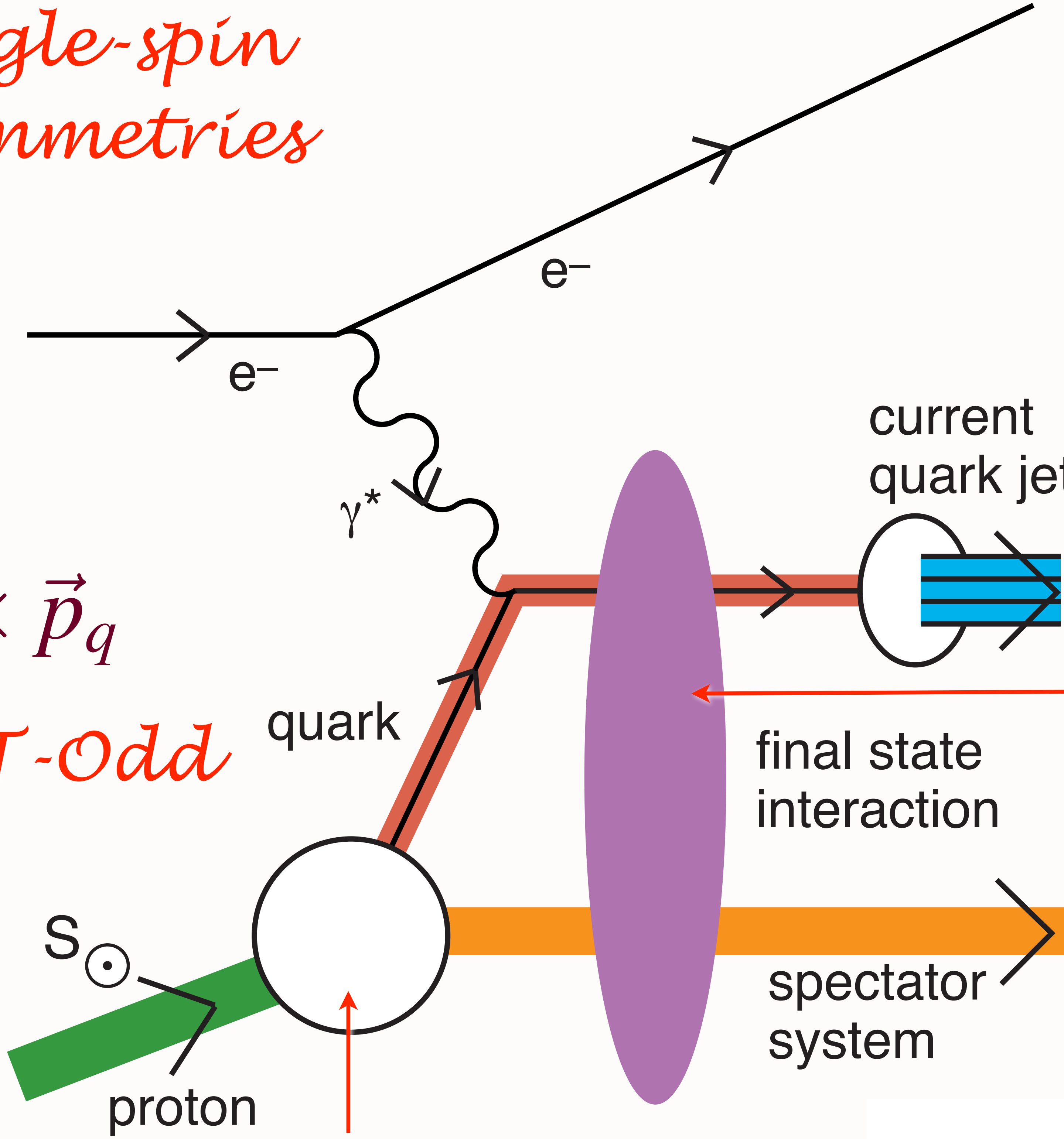
Each Fock State

Single-spin asymmetries

Leading Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt, Ji, Yuan. Pasquini, ...



i $\vec{S}_p \cdot \vec{q} \times \vec{p}_q$

Pseudo- T-Odd

QCD S- and P-
Coulomb Phases
--Wilson Line

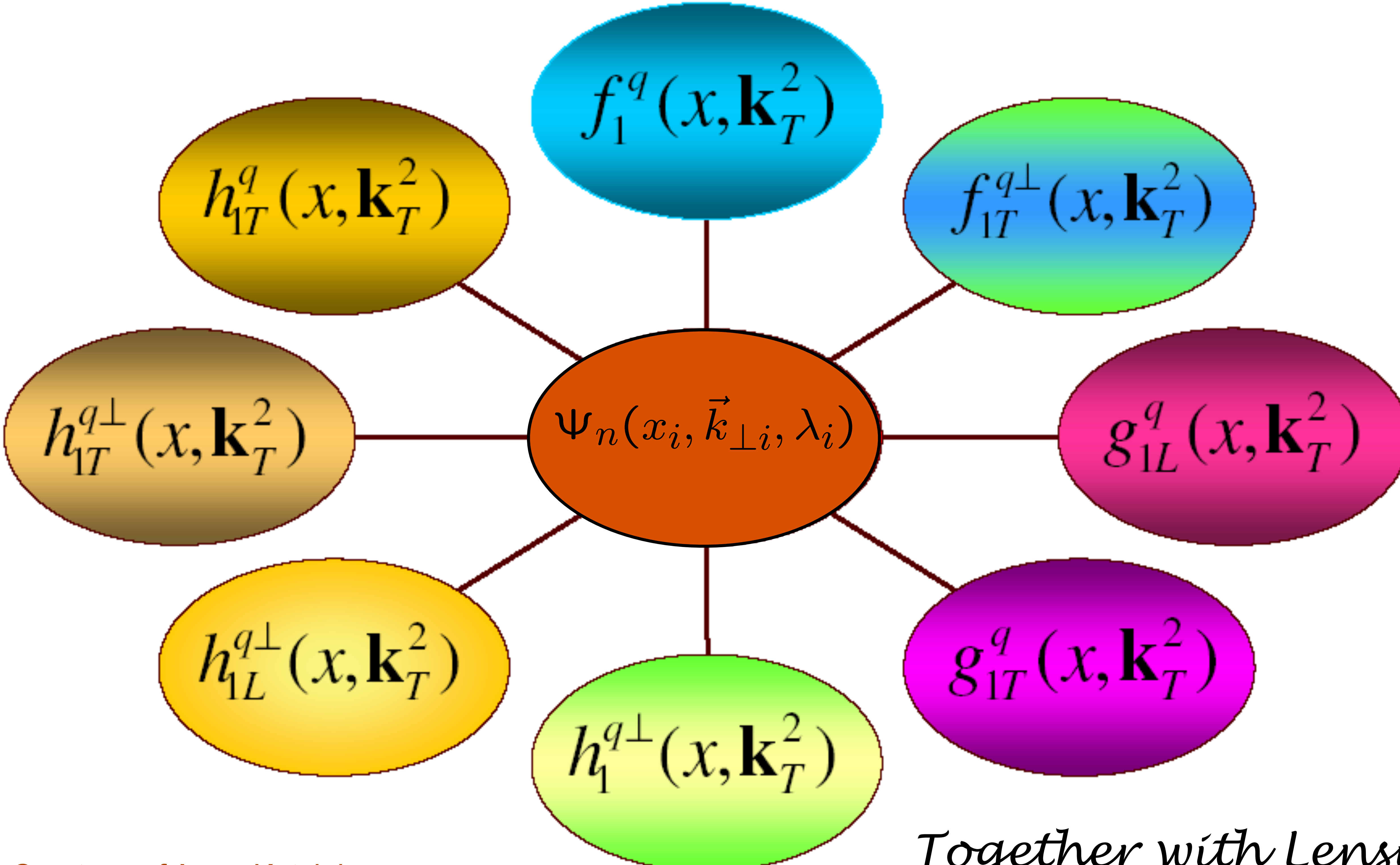
"Lensing Effect"

Light-Front Wavefunction
S and P- Waves!

Leading-Twist
Rescattering
Violates pQCD
Factorization!

Sign reversal in DY!

8 leading-twist spin-$k_\perp$ dependent distribution functions



Together with Lensing

Courtesy of Aram Kotzinian

Example of LFWF representation of GPDs (n => n)

Diehl, Hwang, sjb

$$\frac{1}{\sqrt{1-\zeta}}\frac{\Delta^1 - i\Delta^2}{2M} E_{(n\to n)}(x,\zeta,t)$$

$$= \left(\sqrt{1-\zeta}\right)^{2-n} \sum_{n,\lambda_i} \int \prod_{i=1}^{n} \frac{\mathrm{d}x_i\, \mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3}\, 16\pi^3 \delta\left(1-\sum_{j=1}^{n} x_j\right) \delta^{(2)}\left(\sum_{j=1}^{n} \vec{k}_{\perp j}\right)$$

$$\times\, \delta(x-x_1)\, \psi^{\uparrow *}_{(n)}(x'_i, \vec{k}'_{\perp i}, \lambda_i)\, \psi^{\downarrow}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_i),$$

where the arguments of the final-state wavefunction are given by

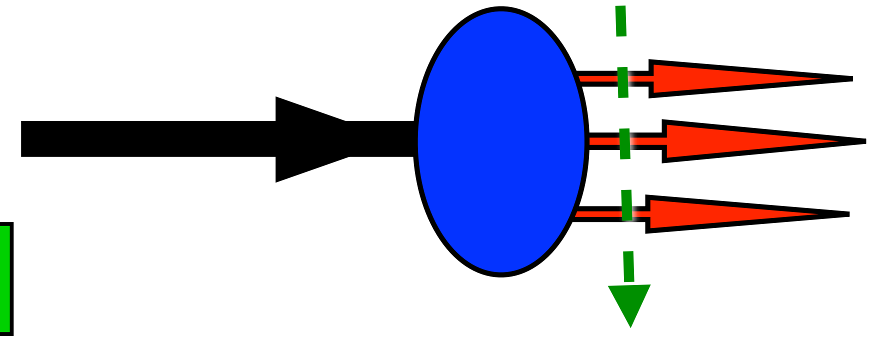
$$x'_1 = \frac{x_1-\zeta}{1-\zeta}, \qquad \vec{k}'_{\perp 1} = \vec{k}_{\perp 1} - \frac{1-x_1}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the struck quark,}$$

$$x'_i = \frac{x_i}{1-\zeta}, \qquad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the spectators } i = 2, \ldots, n.$$

Frame-Independent Hadron Eigenfunctions of the QCD Hamiltonian

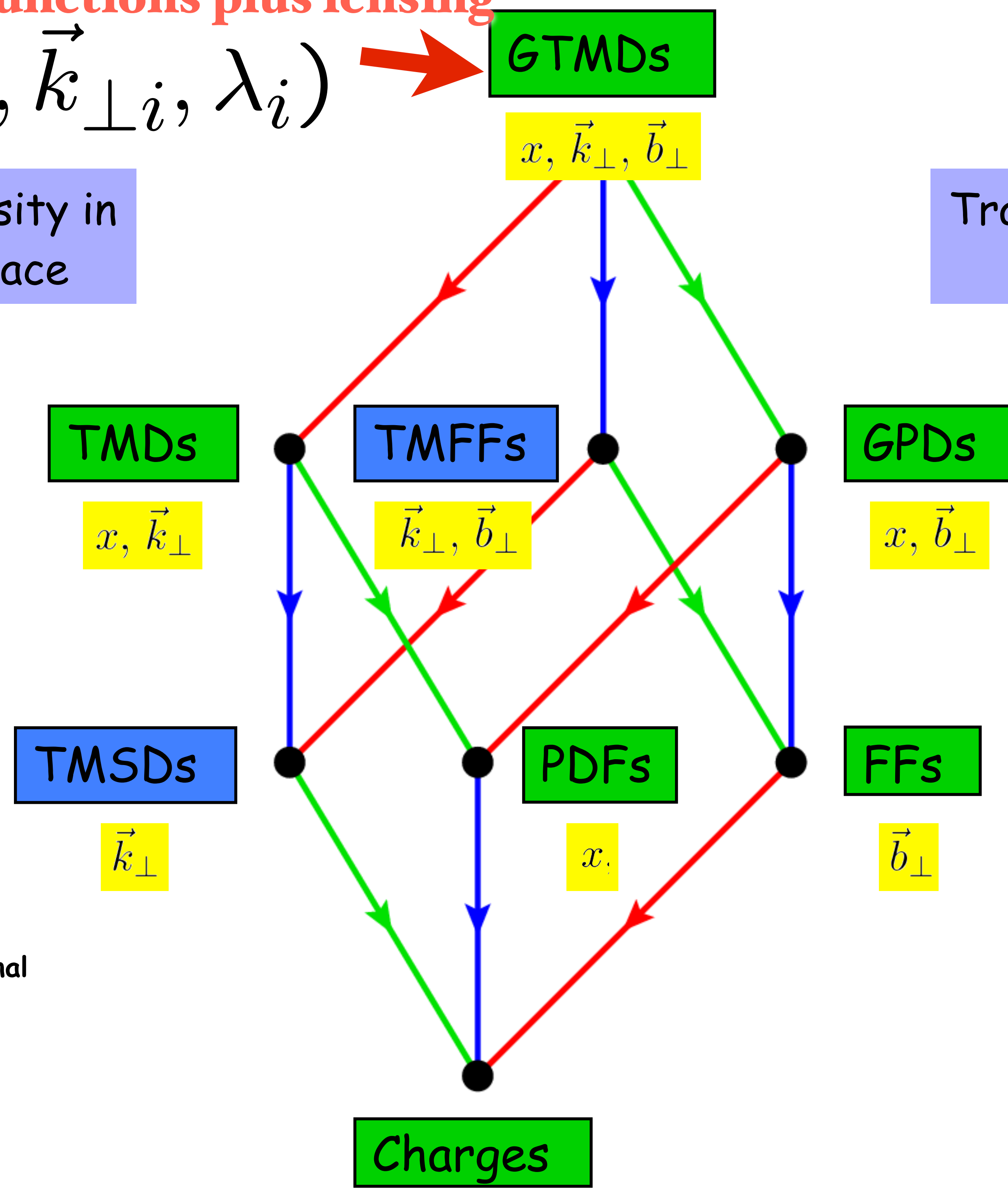
Light-Front Wavefunctions plus lensing

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



Transverse density in momentum space

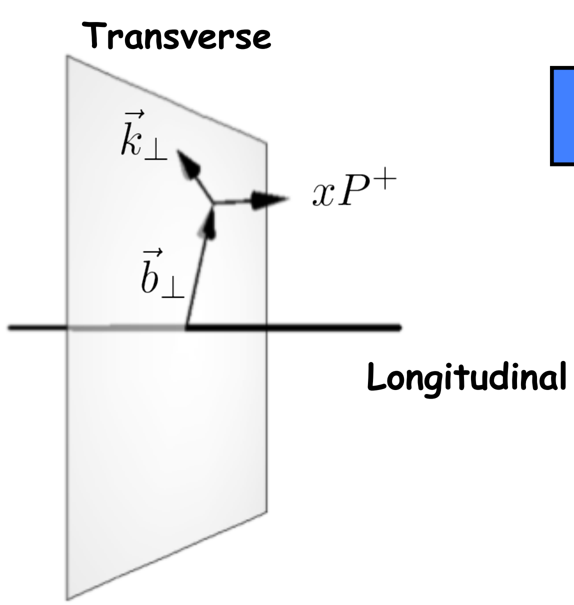
Transverse density in position space

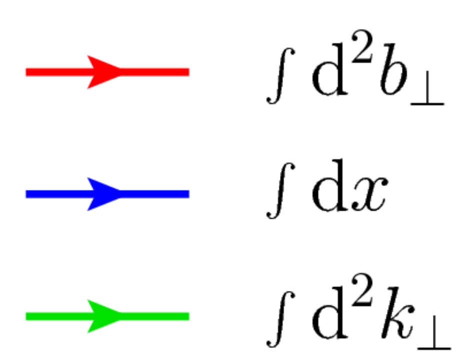


Lorce

$$\vec{k}_\perp \leftrightarrow \vec{z}_\perp$$
$$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$$

$$\xi = 0$$



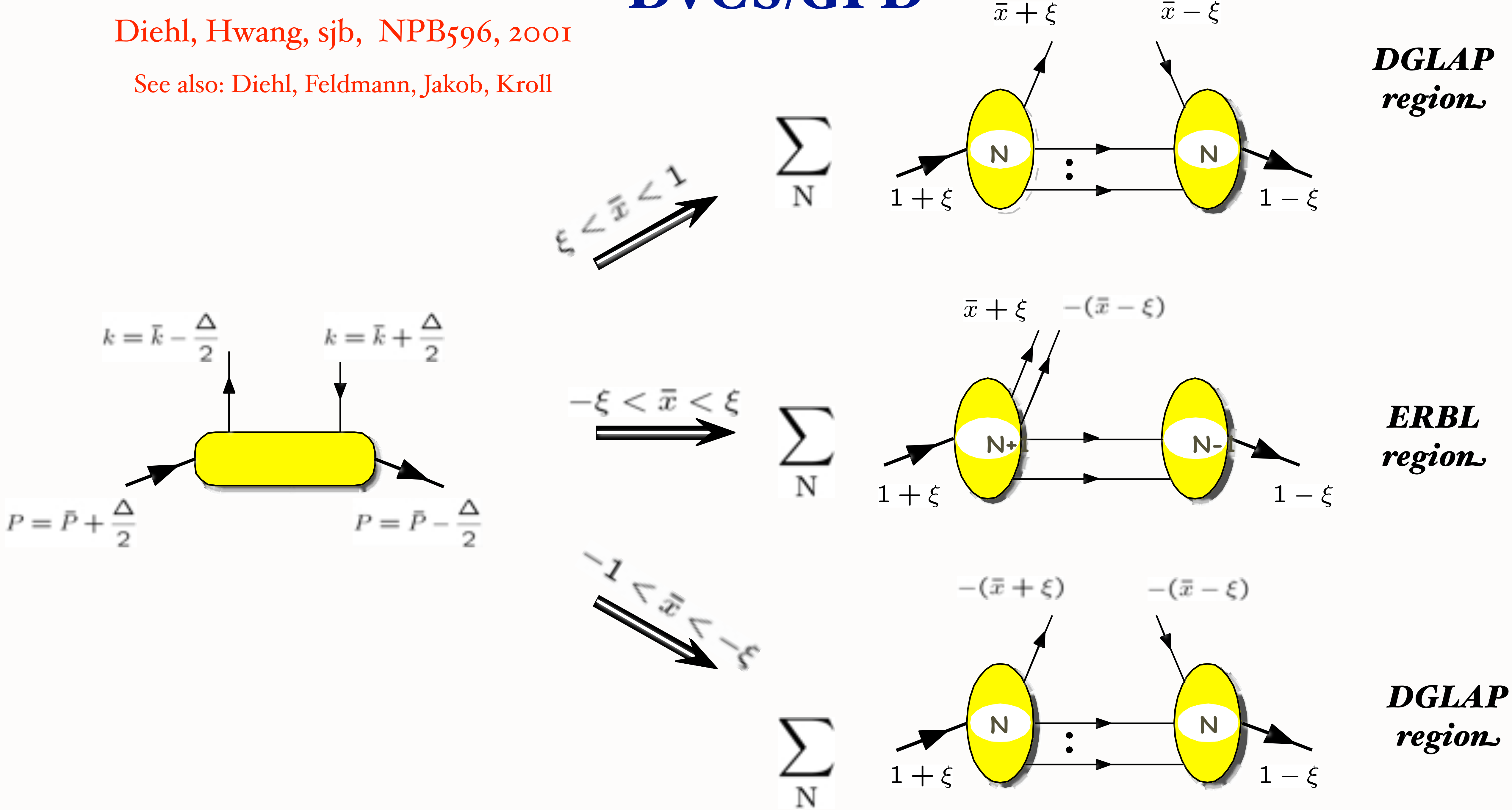


Light-Front Wave Function Overlap Representation

DVCS/GPD

Diehl, Hwang, sjb, NPB596, 2001

See also: Diehl, Feldmann, Jakob, Kroll



Bakker & Ji
Lorce

Link to DIS and Elastic Form Factors

DIS at $\xi = t = 0$

$$H^q(x,0,0) = q(x), \quad -\bar{q}(-x)$$
$$\widetilde{H}^q(x,0,0) = \Delta q(x), \quad \Delta\bar{q}(-x)$$

Form factors (sum rules)

$$\int^1 dx \sum_q \left[H^q(x,\xi,t)\right] = F_1(t) \text{ Dirac f.f.}$$
$$\int^1 dx \sum_q \left[E^q(x,\xi,t)\right] = F_2(t) \text{ Pauli f.f.}$$
$$\int_{-1}^{1} dx\, \widetilde{H}^q(x,\xi,t) = G_{A,q}(t), \quad \int_{-1}^{1} dx\, \widetilde{E}^q(x,\xi,t) = G_{P,q}(t)$$

$$H^q, E^q, \widetilde{H}^q, \widetilde{E}^q(x,\xi,t)$$

Verified using LFWFs

Diehl, Hwang, sjb

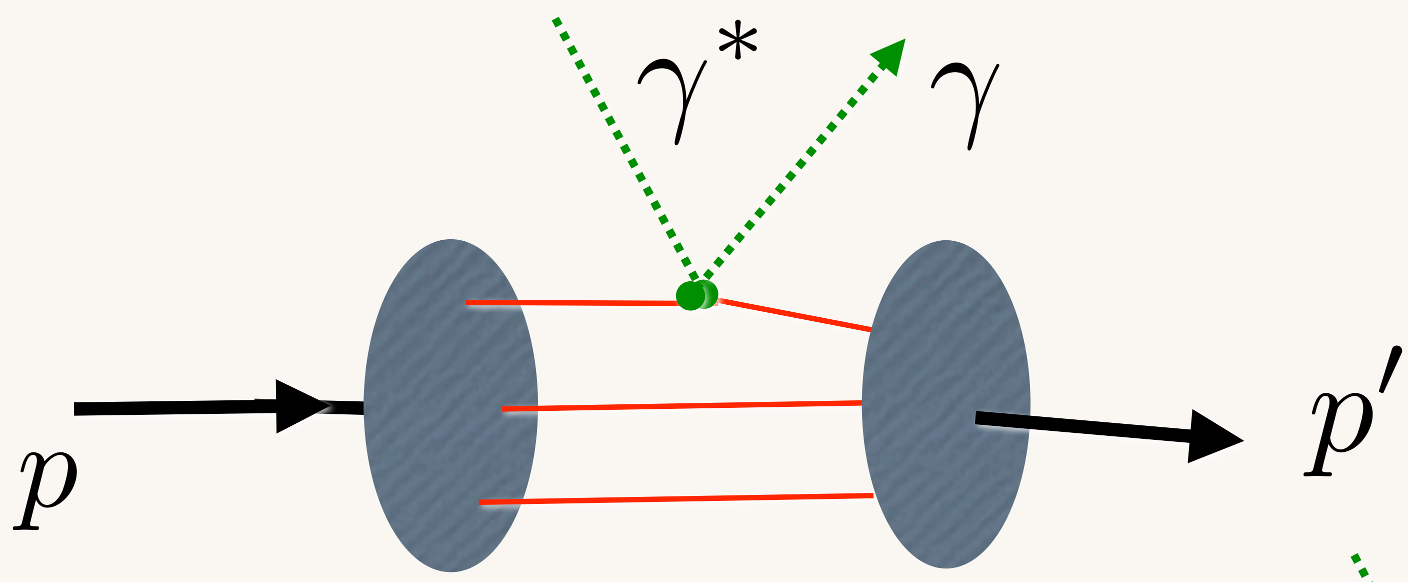
Quark angular momentum (Ji's sum rule)

$$J^q = \frac{1}{2} - J^G = \frac{1}{2}\int_{-1}^{1} x\,dx\left[H^q(x,\xi,0) + E^q(x,\xi,0)\right]$$

X. Ji, Phy.Rev.Lett.78,610(1997)

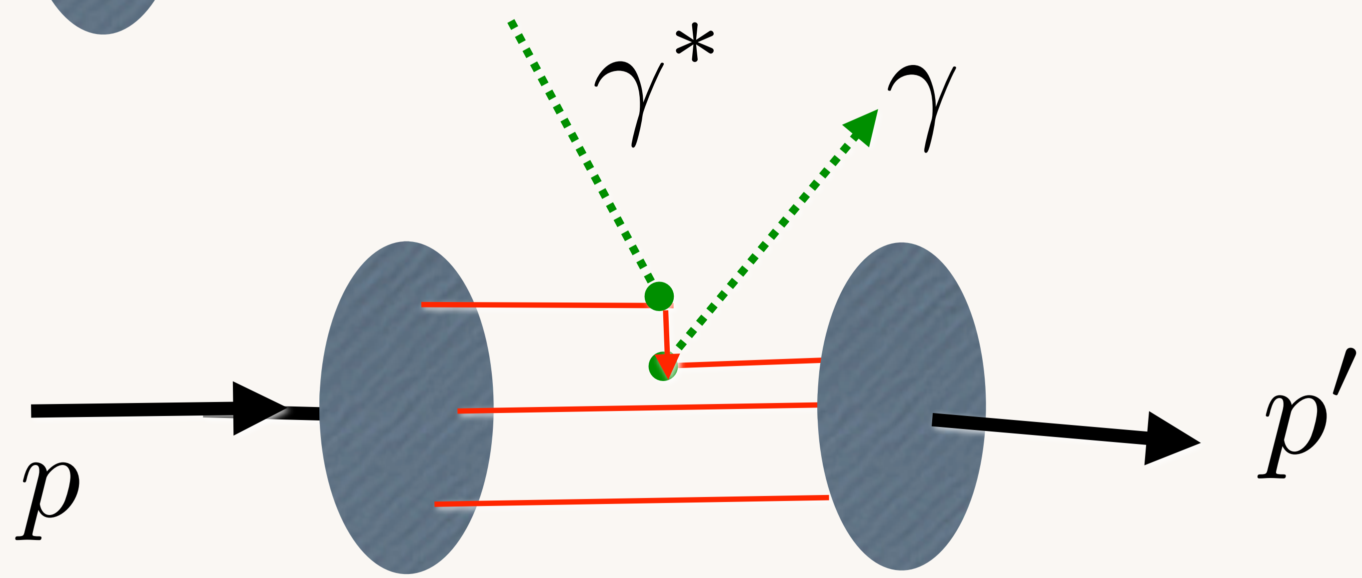
J=0 Fixed Pole Contribution to DVCS

- J=0 fixed pole -- direct test of QCD locality -- from seagull or instantaneous contribution to Feynman propagator



Szczepaniak, Llanes-Estrada, sjb

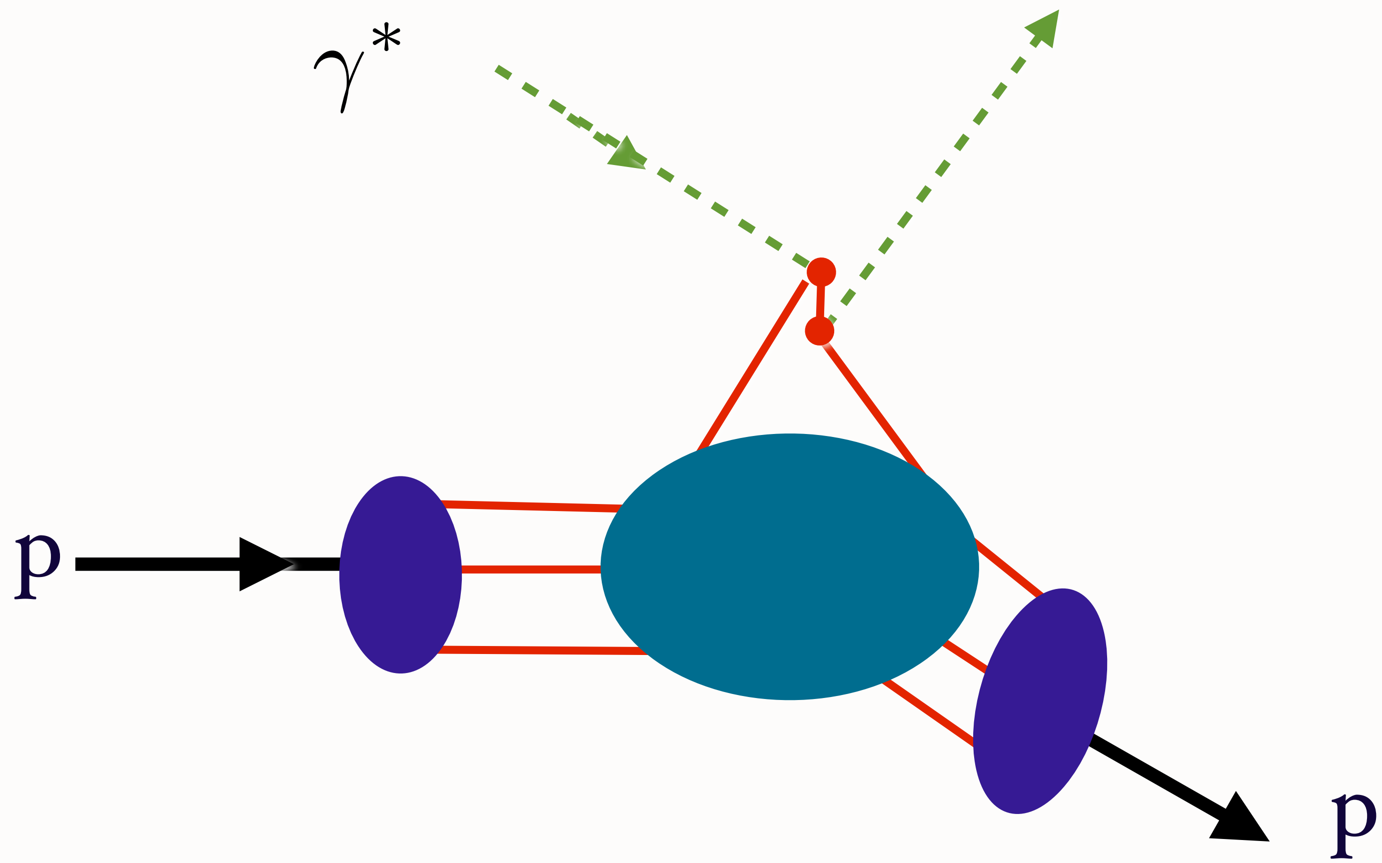
Close, Gunion, sjb



Real amplitude, independent of Q^2 at fixed t

Deeply Virtual Compton Scattering

$$\gamma^* p \to \gamma p$$



Seagull interaction (instantaneous quark exchange or Z-graph)

$$s >> -t, Q^2 >> \Lambda^2_{QCD}$$

Hard Reggeon Domain

$$T(\gamma^*(q)p \to \gamma(k) + p) \sim \epsilon \cdot \epsilon' \sum_R s^{\alpha}_R(t)\beta_R(t)$$

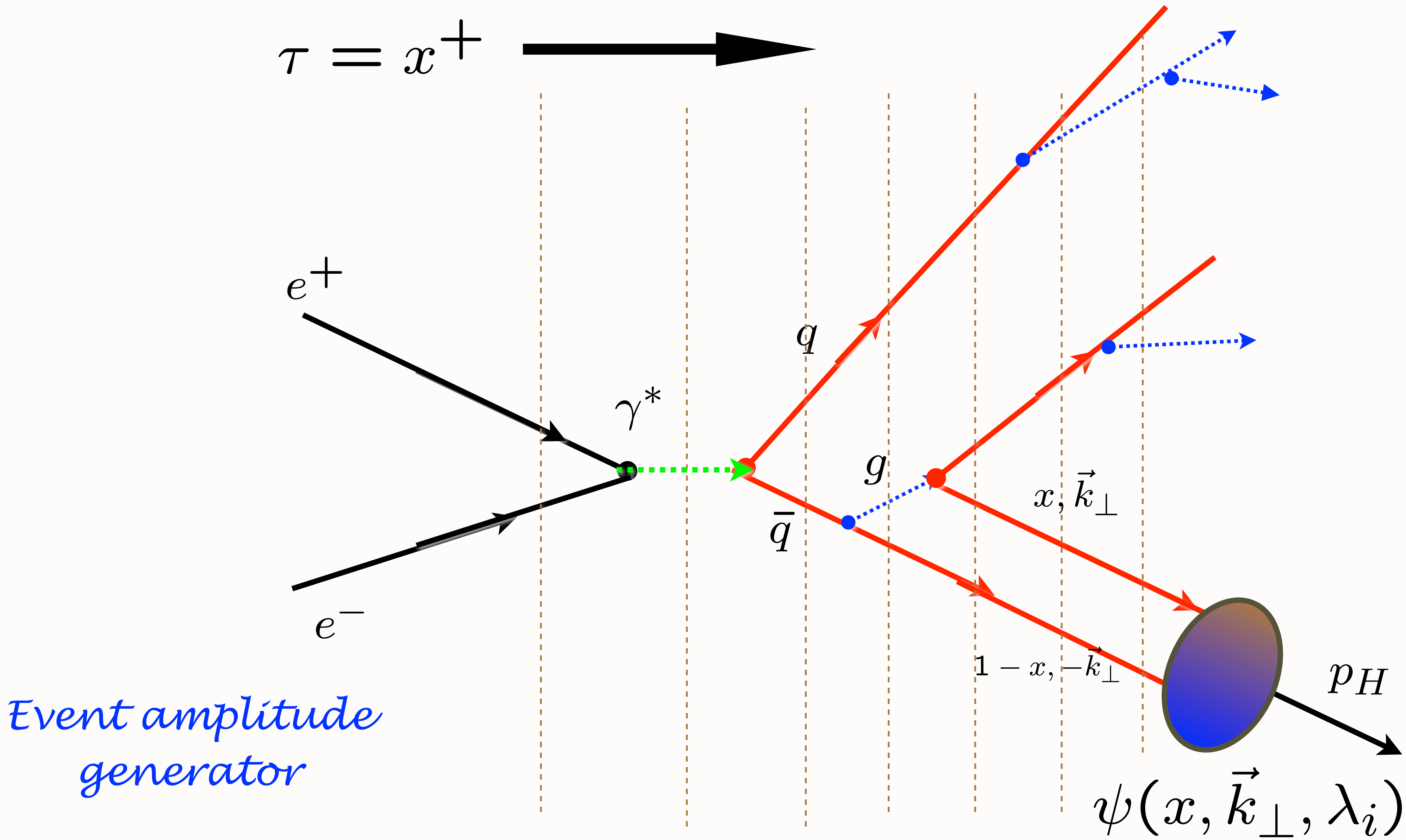
$$\alpha_R(t) \to 0$$

Reflects elementary coupling of two photons to quarks

$$\beta_R(t) \sim \frac{1}{t^2}$$

$$\frac{d\sigma}{dt} \sim \frac{1}{s^2}\frac{1}{t^4} \sim \frac{1}{s^6} \text{ at fixed } \frac{Q^2}{s}, \frac{t}{s}$$

Hadronization at the Amplitude Level

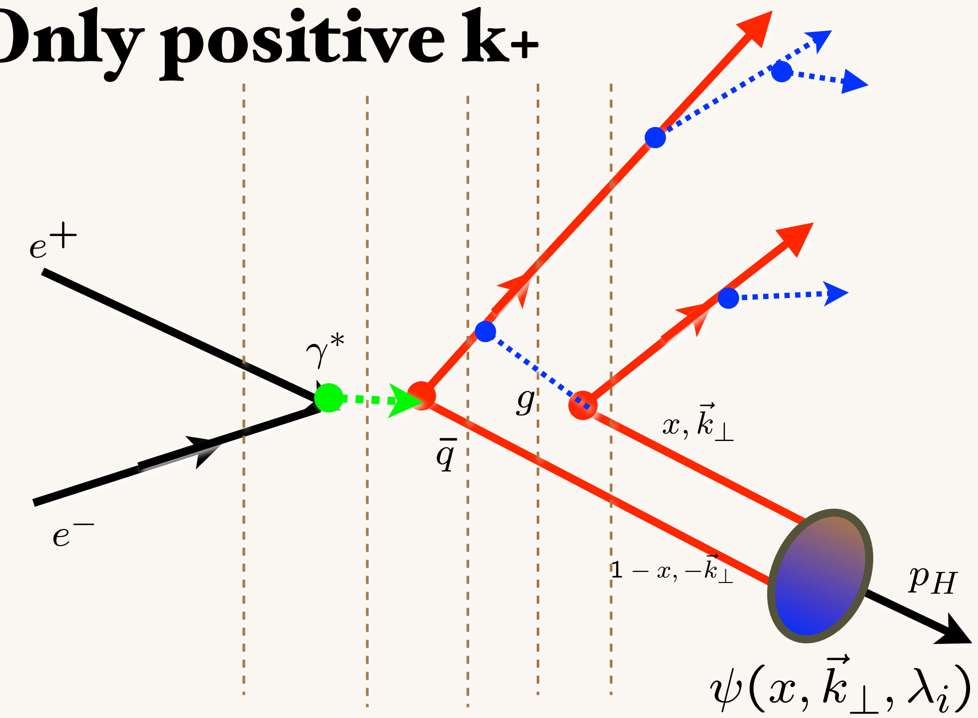


Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Off -Shell T-Matrix

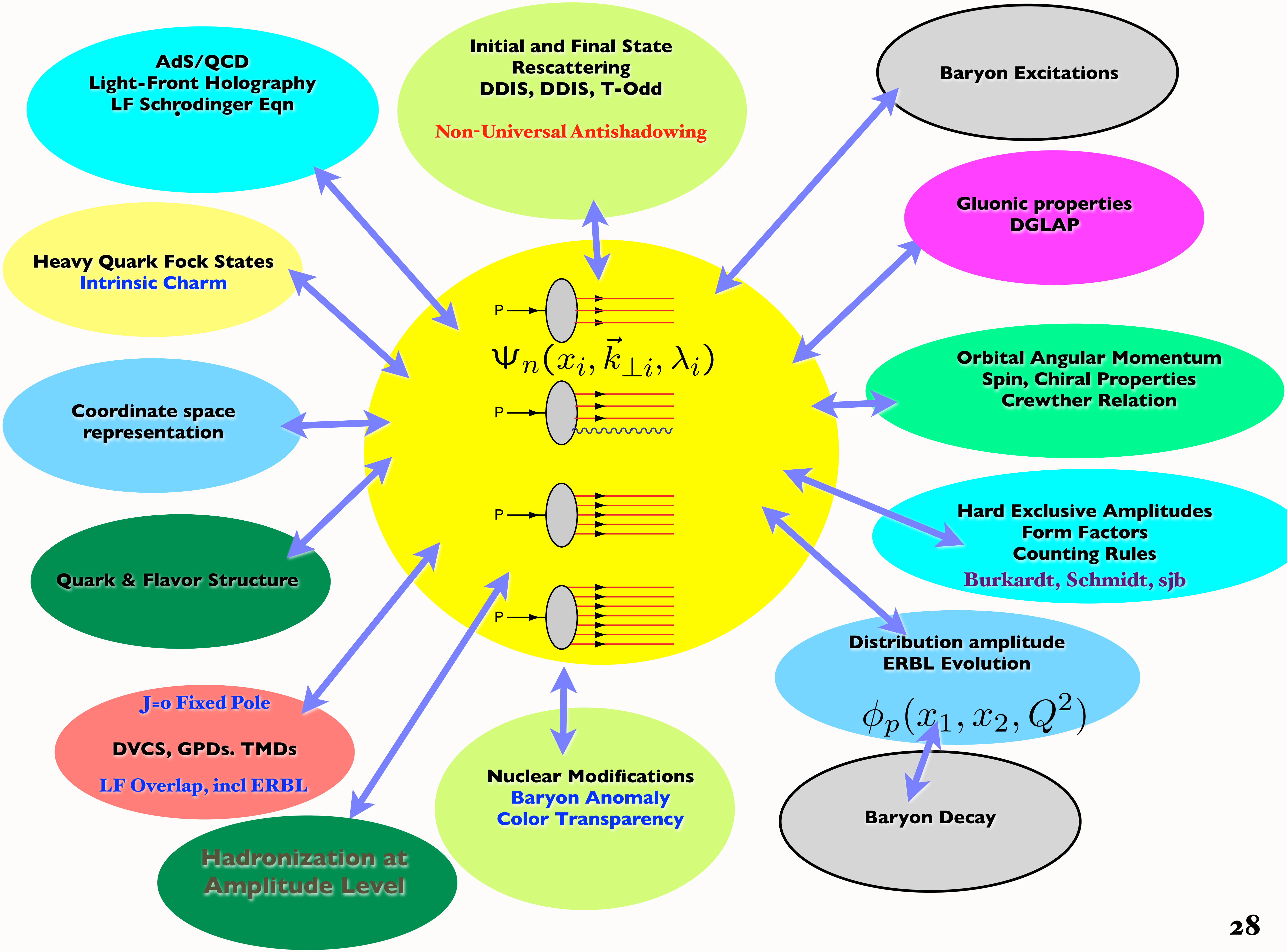
Event amplitude generator

- **Quarks and Gluons Off-Shell**
- **LFPth: Minimal Time-Ordering Diagrams-Only positive k+**
- **J^z Conservation at every vertex**
- **Frame-Independent**
- **Cluster Decomposition** Chueng Ji, sjb
- **"History"-Numerator structure universal**
- **Renormalization- alternate denominators**
- **LFWF takes Off-shell to On-shell**
- **Tested in QED: g-2 to three loops**



Roskies, Suaya, sjb

QCD and LF Hadron Wavefunctions



$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int})\ |\Psi >= E\ |\Psi >$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S},\vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r,S,\ell)]\ \psi(r) = E\ \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \to V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

Bohr Spectrum

Semiclassical first approximation to QED

$$H^{LF}_{QCD}$$

QCD Meson Spectrum

$$(H^0_{LF} + H^I_{LF})|\Psi >= M^2|\Psi >$$

Coupled Fock states

$$[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\rm eff}]\ \psi_{LF}(x, \vec{k}_\perp) = M^2\ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\zeta^2 = x(1-x) b^2_\perp$$

$$[-\frac{d^2}{d\zeta^2} + \frac{4L^2-1}{\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

Azimuthal Basis ζ, ϕ

$$U(\zeta, S, L) = \kappa^2\zeta^2 + \kappa^2(L + S - 1/2)$$

Confining AdS/QCD potential

Semiclassical first approximation to QCD

de Teramond, sjb

Derivation of the Light-Front Radial Schrodinger Equation directly from LF QCD

$$\mathcal{M}^2 = \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left|\psi(x,\vec{k}_\perp)\right|^2 + \text{interactions}$$

$$= \int_0^1 \frac{dx}{x(1-x)} \int d^2\vec{b}_\perp \, \psi^*(x,\vec{b}_\perp) \left(-\vec{\nabla}^2_{\vec{b}_{\perp\ell}}\right) \psi(x,\vec{b}_\perp) + \text{interactions.}$$

Change variables $(\vec{\zeta},\varphi), \ \vec{\zeta} = \sqrt{x(1-x)}\vec{b}_\perp: \quad \nabla^2 = \frac{1}{\zeta}\frac{d}{d\zeta}\left(\zeta\frac{d}{d\zeta}\right) + \frac{1}{\zeta^2}\frac{\partial^2}{\partial\varphi^2}$

$$\mathcal{M}^2 = \int d\zeta \, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}}$$

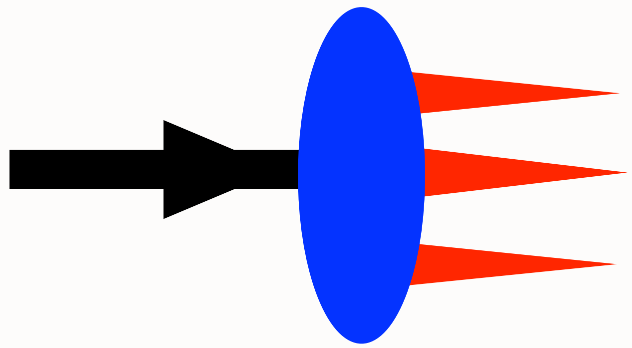
$$+ \int d\zeta \, \phi^*(\zeta) U(\zeta) \phi(\zeta)$$

$$= \int d\zeta \, \phi^*(\zeta)\left(-\frac{d^2}{d\zeta^2} + \boxed{\frac{4L^2-1}{4\zeta^2}} + U(\zeta)\right)\phi(\zeta)$$

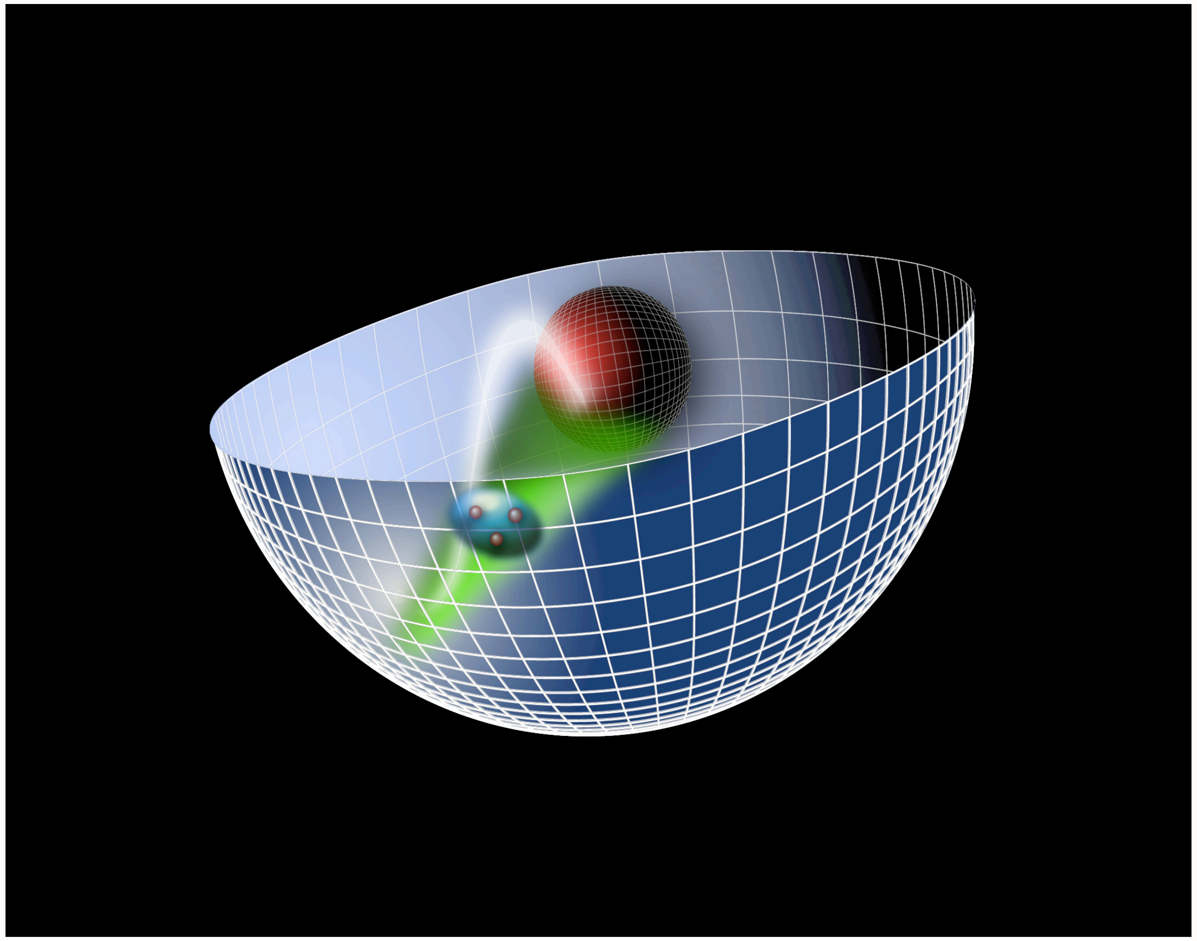
Light-Front Holography and Non-Perturbative QCD

Goal:
Use AdS/QCD duality to construct a first approximation to QCD

Hadron Spectrum
Light-Front Wavefunctions,
Running coupling in IR

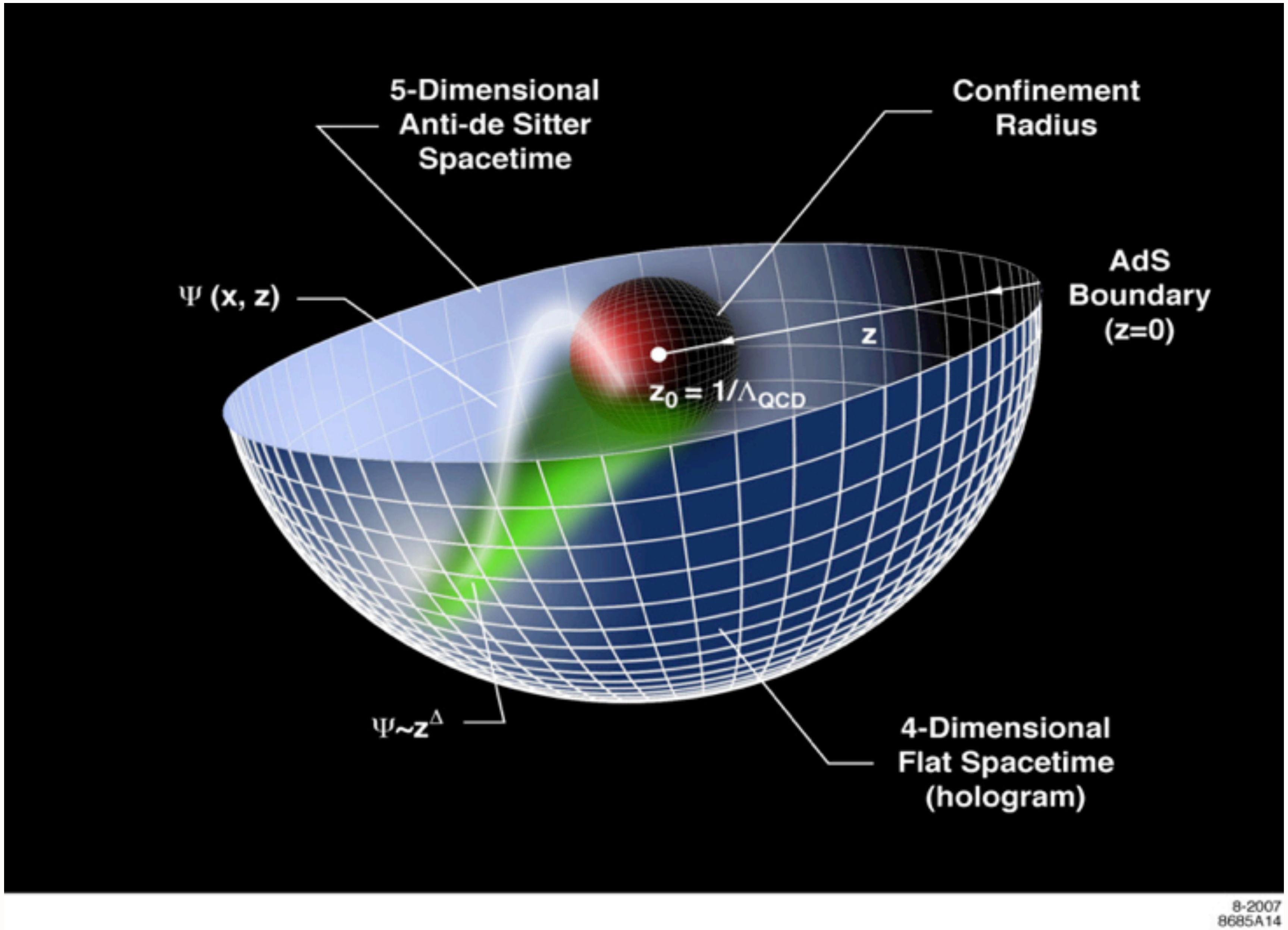


$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



in collaboration with Guy de Teramond

Central problem for strongly-coupled gauge theories



Changes in physical length scale mapped to evolution in the 5th dimension z

Light-Front Holography

in collaboration with Guy de Teramond

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).

- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model)
  - Erlich, Karch, Katz, Son, Stephanov

Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \to \left(R^2/z^2\right) \eta_{\ell m}$.

- Action for massive scalar modes on AdS$_{d+1}$:

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} \, \tfrac{1}{2} \left[g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \to (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left(\sqrt{g} \, g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along x^μ-coordinates , $\Phi_P(x, z) = e^{-iP \cdot x} \, \Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$:

$$\boxed{\left[z^2 \partial_z^2 - (d-1) z \, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.}$$

- Solution: $\Phi(z) \to z^\Delta$ as $z \to 0$,

$$\Phi(z) = C z^{d/2} J_{\Delta - d/2}(z \mathcal{M}) \qquad \Delta = \tfrac{1}{2} \left(d + \sqrt{d^2 + 4\mu^2 R^2} \right).$$

$$\Delta = 2 + L \qquad d = 4 \qquad (\mu R)^2 = L^2 - 4$$

Let $\Phi(z) = z^{3/2}\phi(z)$

AdS Schrodinger Equation for bound state of two scalar constituents:

$$[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2}]\phi(z) = \mathcal{M}^2\phi(z)$$

$L = L^z$: light-front orbital angular momentum

Derived from variation of Action in AdS_5

Hard wall model: truncated space

$$\phi(\mathbf{z} = \mathbf{z}_0 = \frac{\mathbf{1}}{\Lambda_\mathbf{c}}) = \mathbf{0}.$$

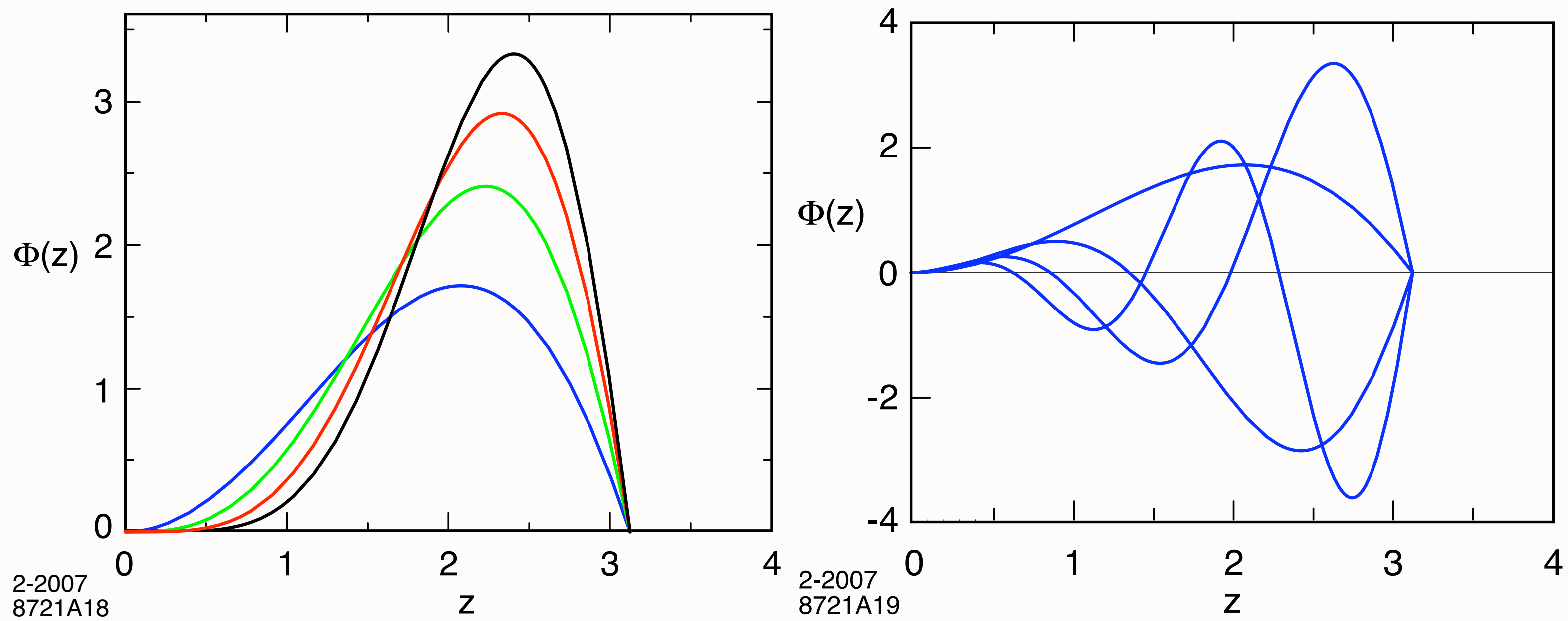


Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{\rm QCD}$ = 0.32 GeV .

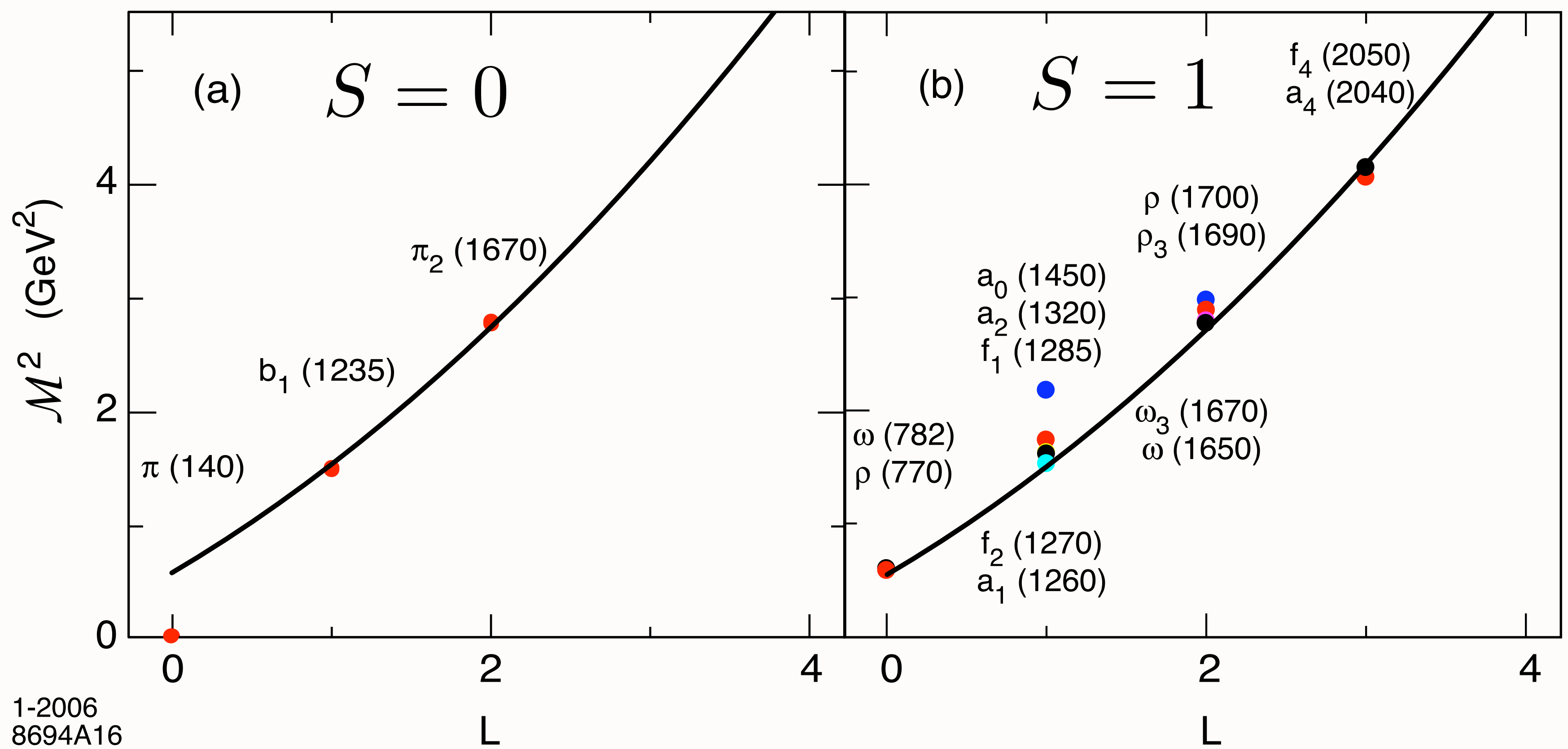


Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV

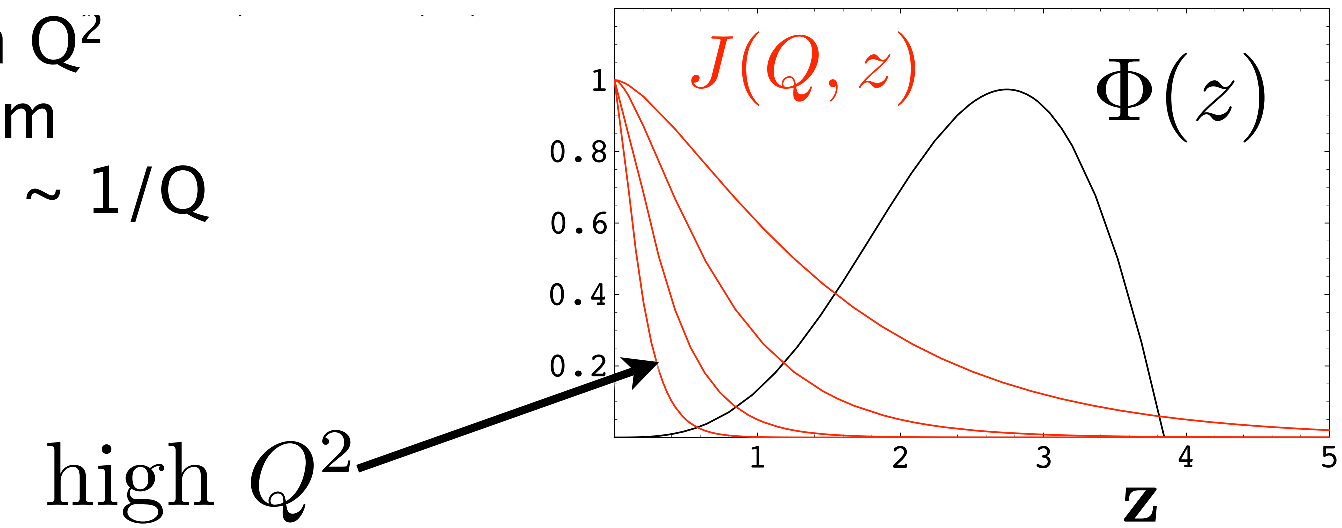
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

High Q^2
from
small z ~ 1/Q



Polchinski, Strassler
de Teramond, sjb

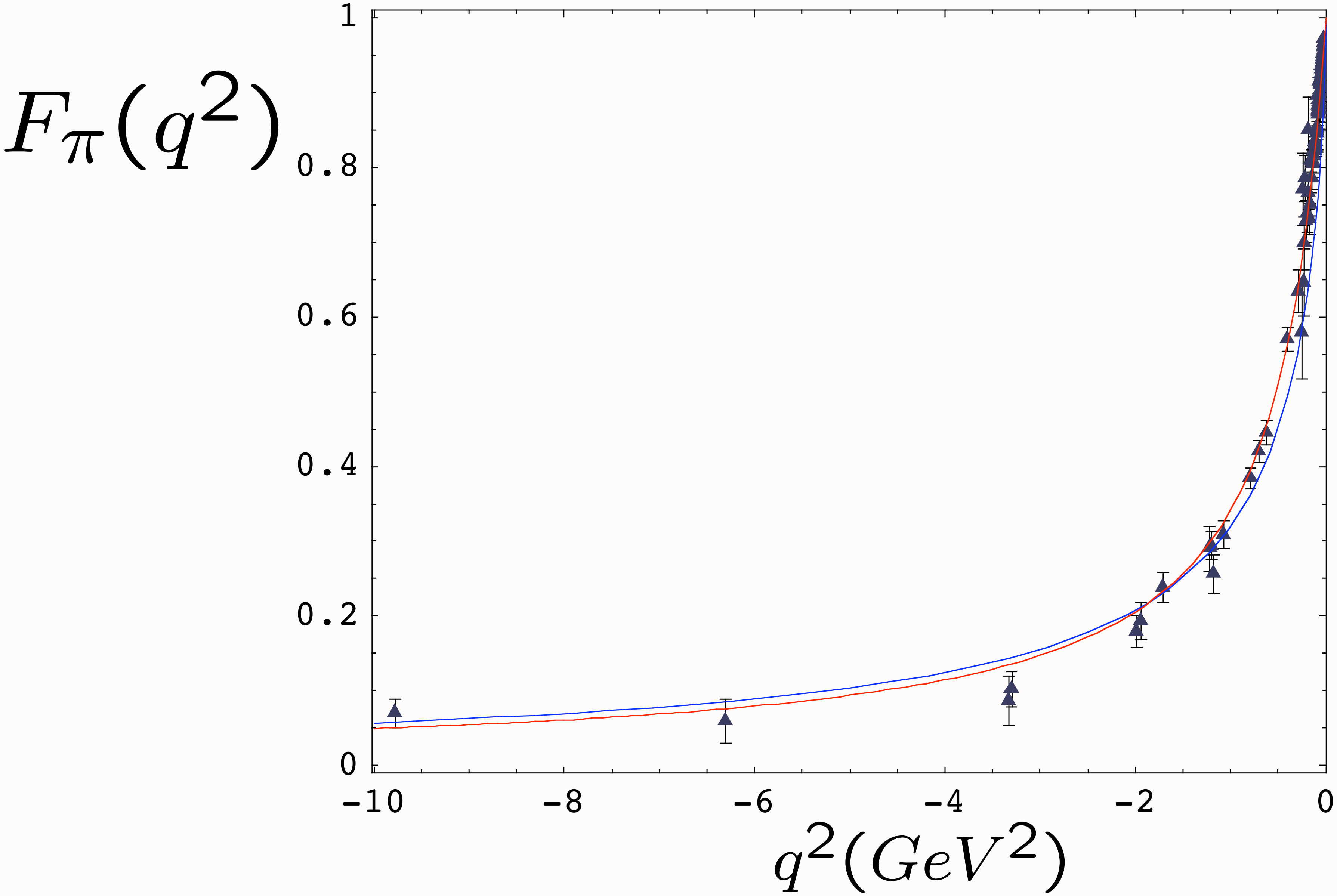
Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, Φ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

Dimensional Quark Counting Rules:
General result from
AdS/CFT and Conformal Invariance

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^n \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Spacelike pion form factor from AdS/CFT



Data Compilation
Baldini, Kloe and Volmer

Soft Wall: Harmonic Oscillator Confinement

Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant

de Teramond, sjb
See also: Radyushkin

- Nonconformal metric dual to a confining gauge theory

$$ds^2 = \frac{R^2}{z^2}\, e^{\varphi(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

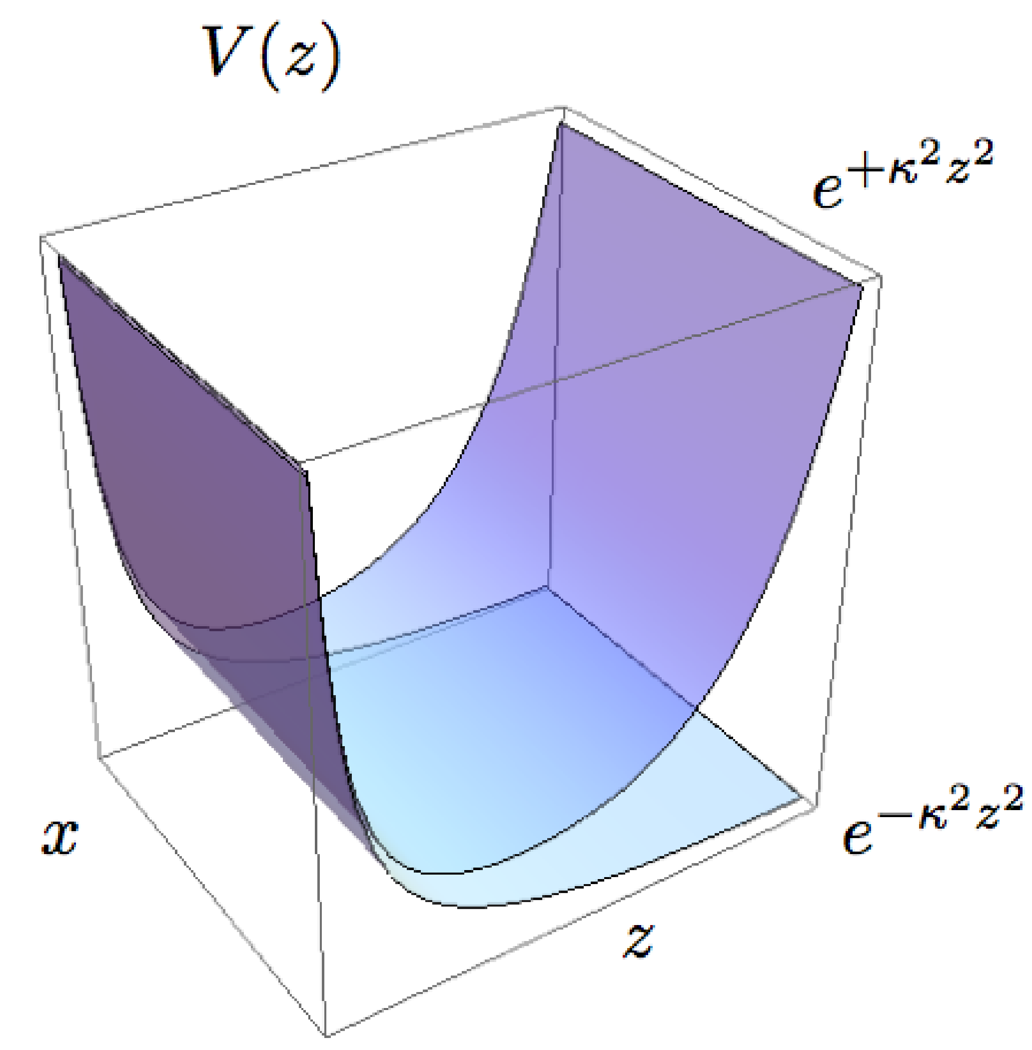
where $\varphi(z) \to 0$ at small z for geometries which are asymptotically AdS_5

- Gravitational potential energy for object of mass m

$$V = mc^2 \sqrt{g_{00}} = mc^2 R\, \frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm\kappa^2 z^2)$

- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z \rangle \sim 1/\kappa$



- de Teramond, sjb

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2}+\frac{4L^2-1}{4z^2}+U(z)\right]\phi(z)=\mathcal{M}^2\phi(z)$$

$$U(z)=\kappa^4 z^2+2\kappa^2(L+S-1)$$

Derived from variation of Action : Dilaton-Modified AdS_5

$$\mathcal{S}\to\mathcal{S}\Phi(z)=\mathcal{S}e^{+\kappa^2 z^2}$$

Positive-sign dilaton

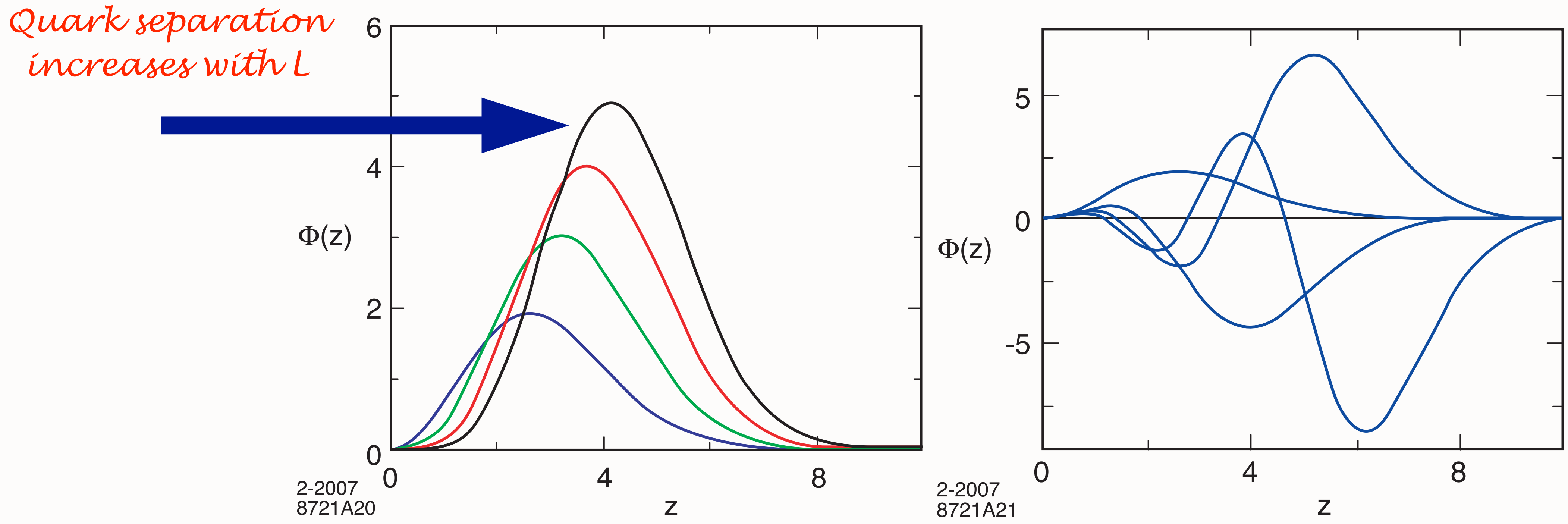
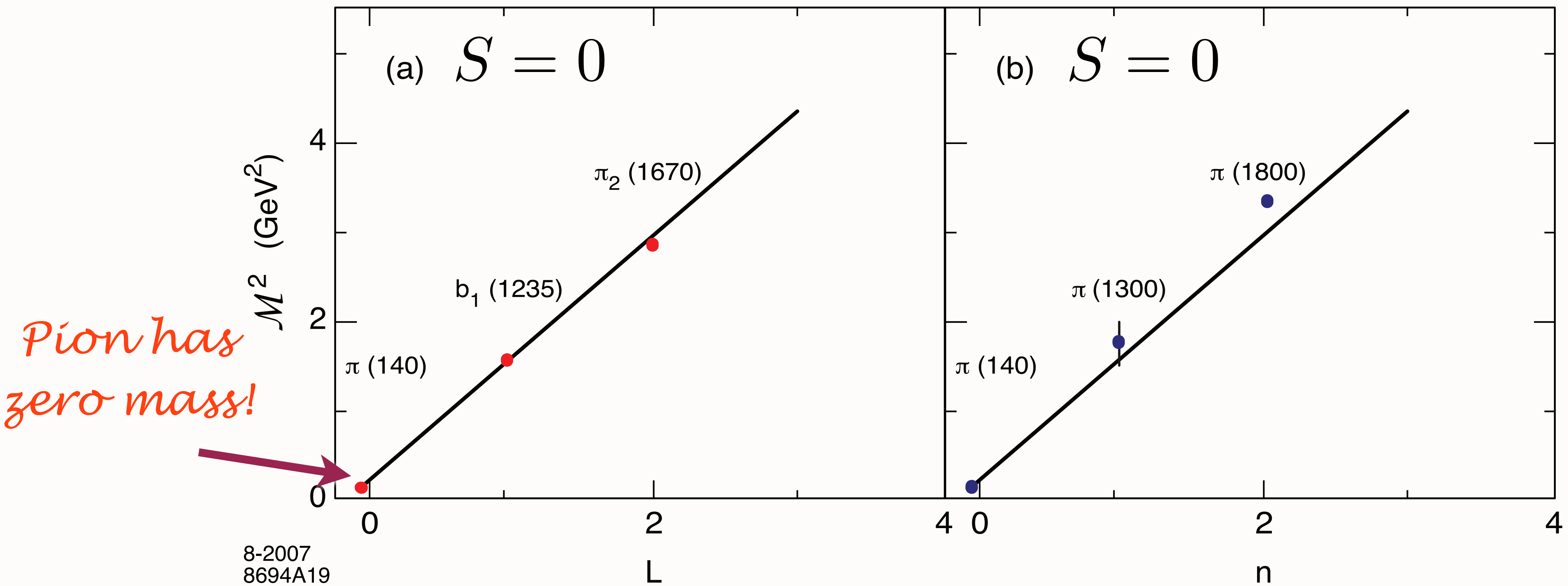


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Soft Wall Model



Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Pion mass automatically zero!

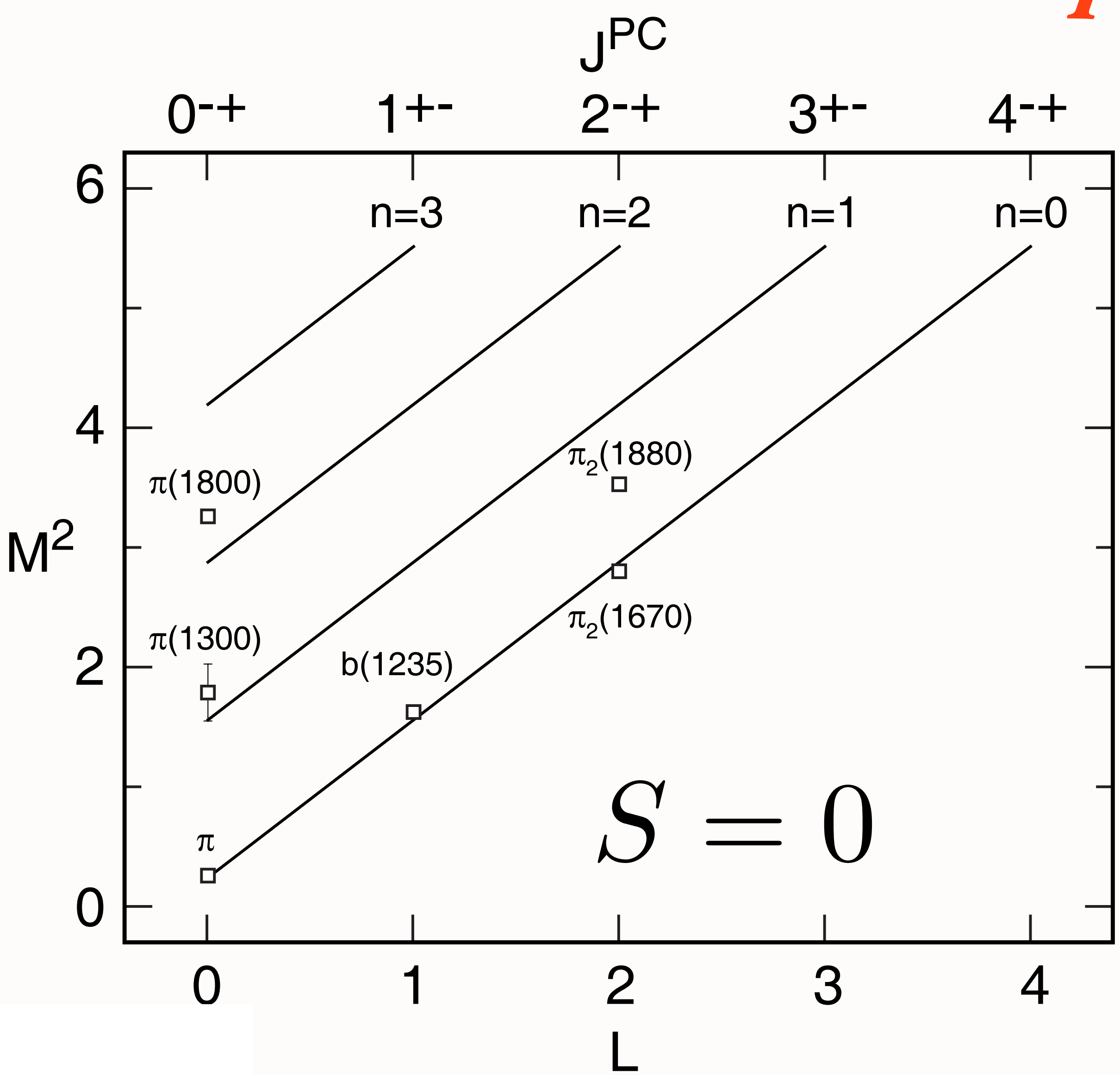
$$m_q = 0$$

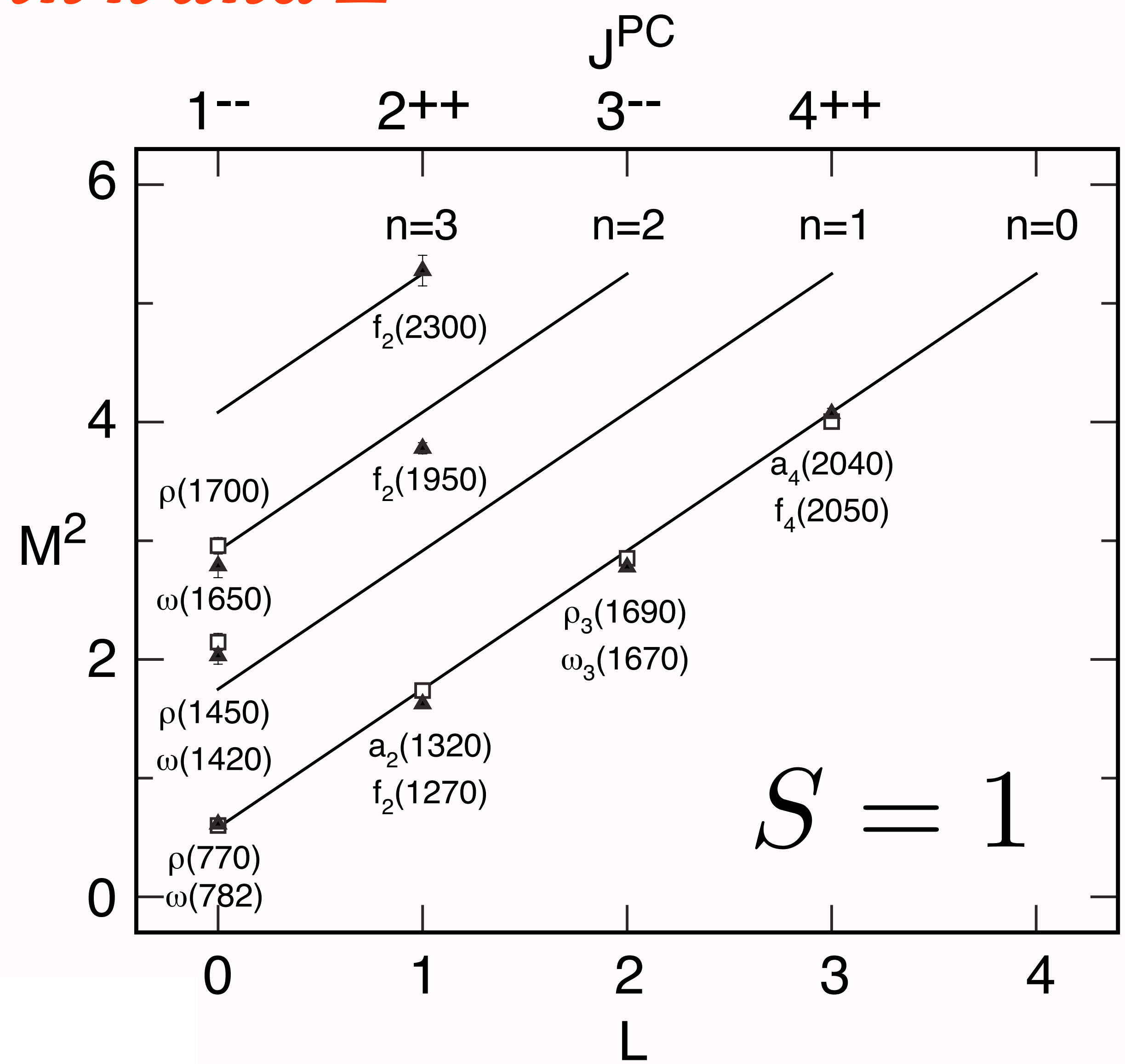
Bosonic Modes and Meson Spectrum

$$\mathcal{M}^2 = 4\kappa^2(n + J/2 + L/2) \to 4\kappa^2(n + L + S/2)$$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

Same slope in n and L





Regge trajectories for the π ($\kappa = 0.6$ GeV) and the $I = 1$ ρ-meson and $I = 0$ ω-meson families ($\kappa = 0.54$ GeV)

General-Spin Hadrons

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

- Substituting in the AdS scalar wave equation for Φ

$$\left[z^2 \partial_z^2 - \left(3 - 2J - 2\kappa^2 z^2\right) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0$$

- Upon substitution $z \to \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2/2}\, \Phi_J(\zeta)$$

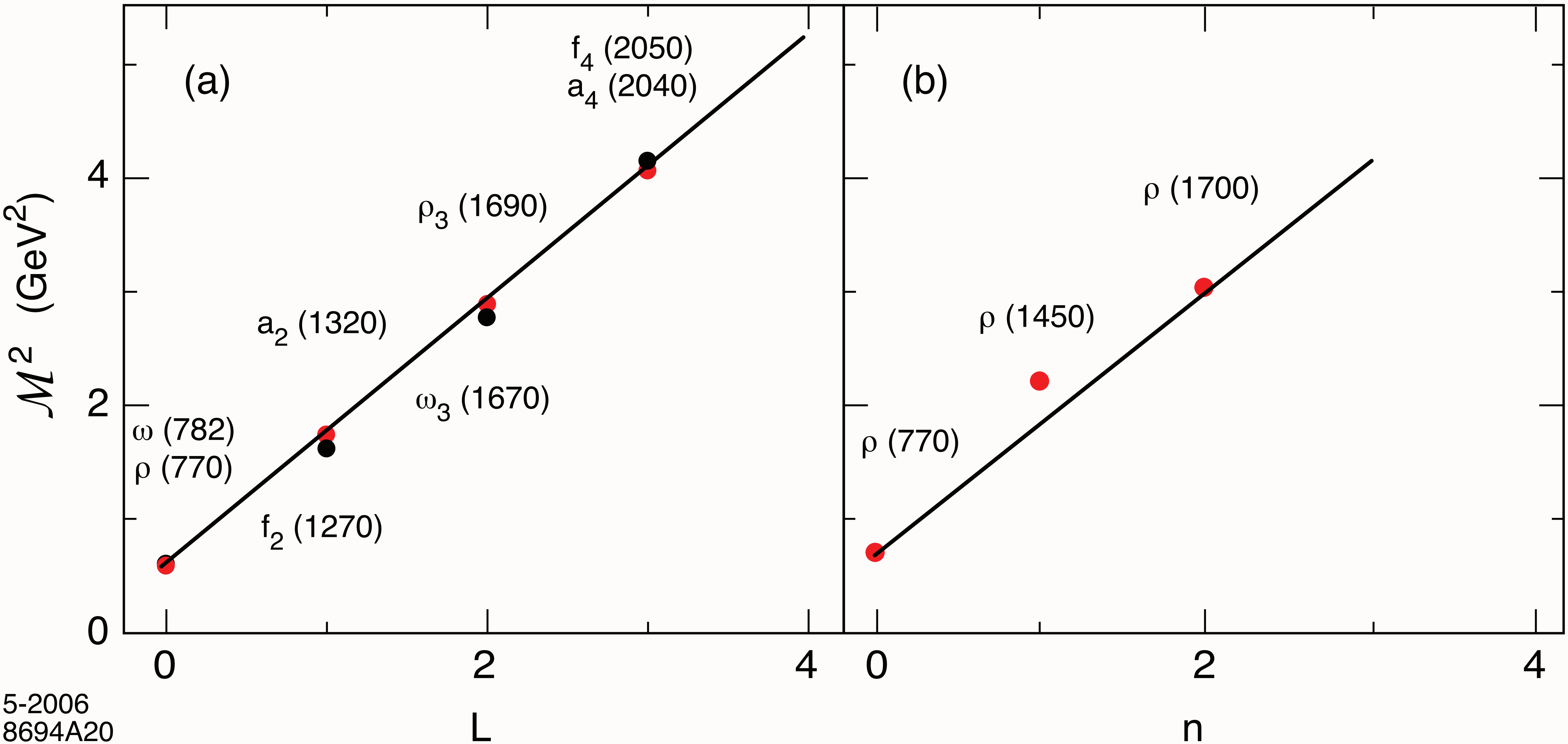
we find the LF wave equation

$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)\right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}}$$

with $(\mu R)^2 = -(2 - J)^2 + L^2$

$$\mathcal{M}^2 = 2\kappa^2(2n + 2L + S).$$

$$S = 1$$



- de Teramond, sjb

$$e^{\Phi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

AdS Soft-Wall Schrodinger Equation for bound state of two constituents:

$$[-\frac{d^2}{dz^2} + \frac{4L^2-1}{4z^2} + U(z)]\phi(z) = \mathcal{M}^2 \phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L+S-1)$$

Derived from variation of Action : Dilaton-Modified AdS_5

Matches the LF QCD Schrodinger Equation !

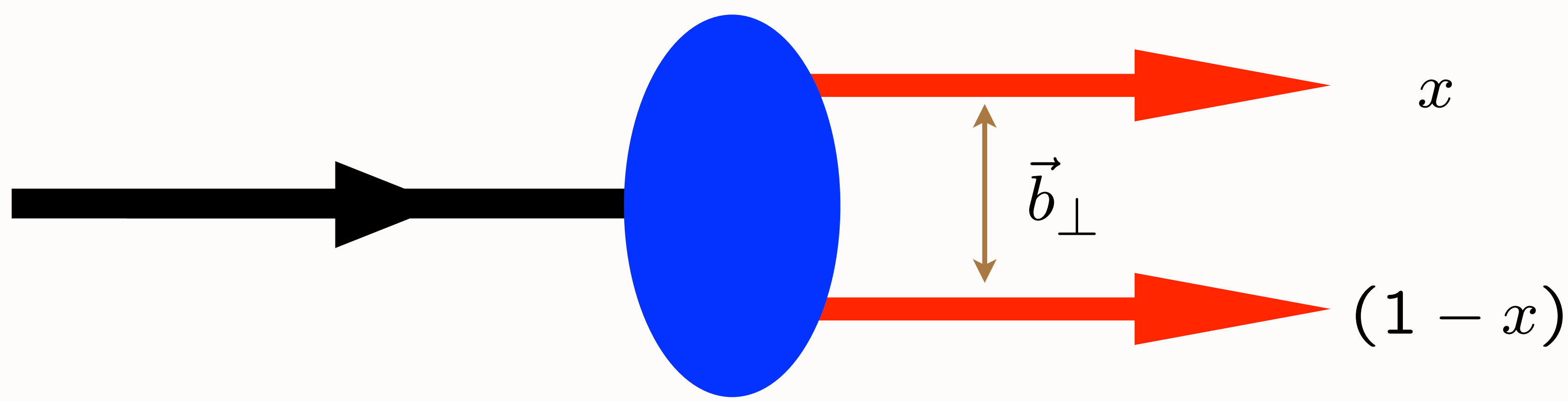
$$[-\frac{d^2}{d\zeta^2} + \frac{4L^2-1}{\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = \mathcal{M}^2\ \psi_{LF}(\zeta)$$

de Teramond, sjb

LF(3+1) $\longleftrightarrow$ AdS_5

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^{\,2}} \longleftrightarrow z$$



$$\psi(x, \zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Light Front Holography: Identical mapping derived from equality of LF (DYW) and AdS formulas for current matrix elements

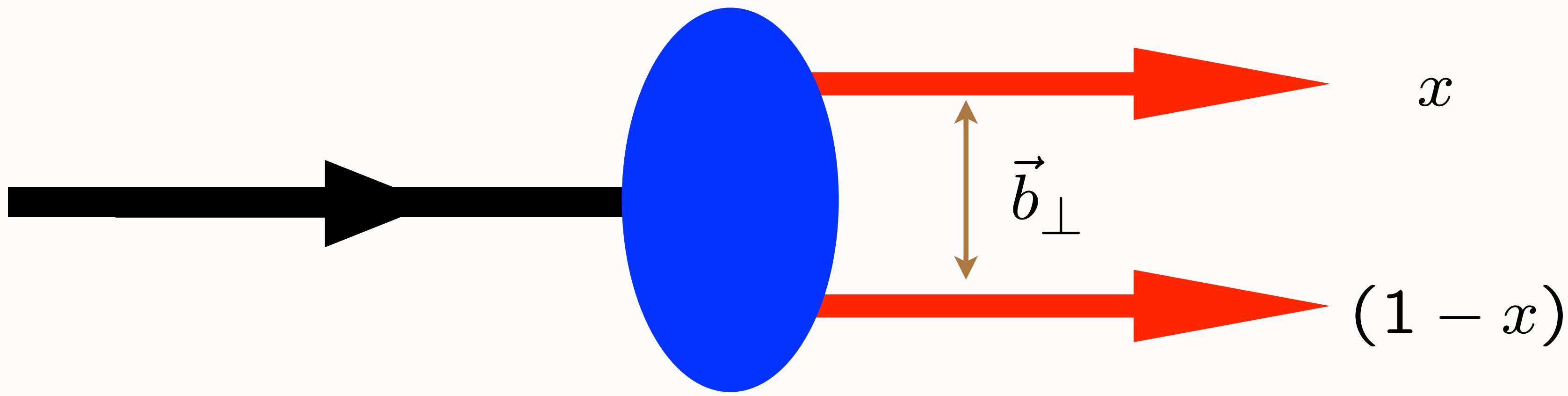
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + \frac{4L^2-1}{\zeta^2} + U(\zeta, S, L)\right] \psi_{LF}(\zeta) = M^2 \ \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp .$$



$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L+S-1)$$

soft wall confining potential:

G. de Teramond, sjb

Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

$$A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} H(Q^2, z) \left|\Phi_\pi(z)\right|^2,$$

Abidin & Carlson

where $H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ)$

- Use integral representation for $H(Q^2, z)$

$$H(Q^2, z) = 2 \int_0^1 x \, dx \, J_0\left(zQ\sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS gravitational form-factor as

$$A_\pi(Q^2) = 2R^3 \int_0^1 x \, dx \int \frac{dz}{z^3} J_0\left(zQ\sqrt{\frac{1-x}{x}}\right) \left|\Phi_\pi(z)\right|^2$$

- Compare with gravitational form-factor in light-front QCD for arbitrary Q

$$\boxed{\left|\tilde{\psi}_{q\overline{q}/\pi}(x, \zeta)\right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{\left|\Phi_\pi(\zeta)\right|^2}{\zeta^4},}$$

Identical to LF Holography obtained from electromagnetic current

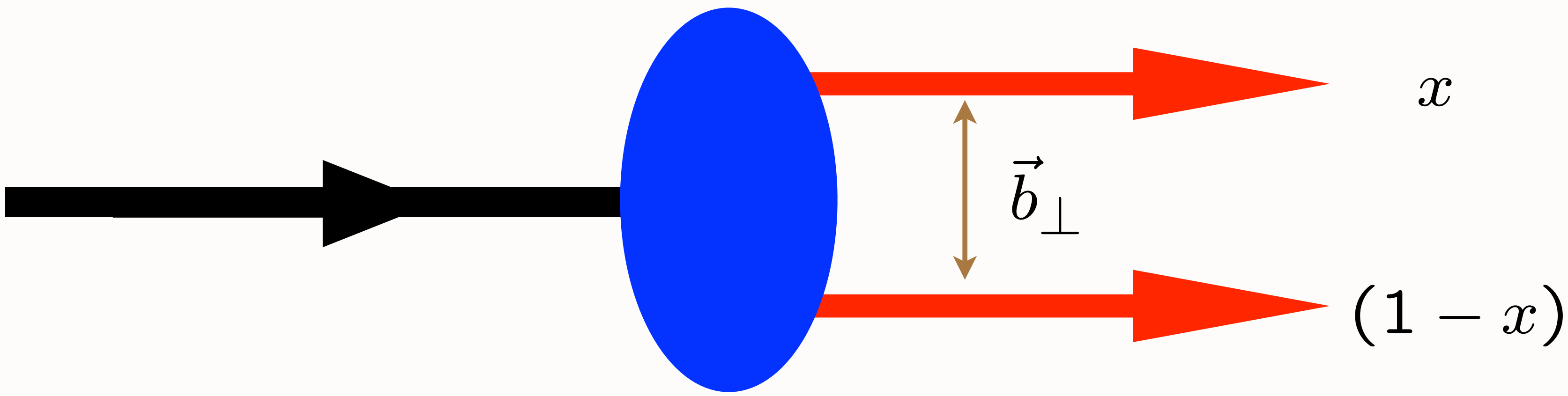
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation

Frame Independent

$$[-\frac{d^2}{d\zeta^2} + \frac{4L^2-1}{\zeta^2} + U(\zeta, S, L)]\, \psi_{LF}(\zeta) = \mathcal{M}^2\, \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp.$$



$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

soft wall confining potential:

G. de Teramond, sjb

Current Matrix Elements in AdS Space (SW)

sjb and GdT
Grigoryan and Radyushkin

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1 + 2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right] J_\kappa(Q, z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

Soft Wall Model

where $U(a, b, c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1}(1+t)^{b-a-1} dt.$$

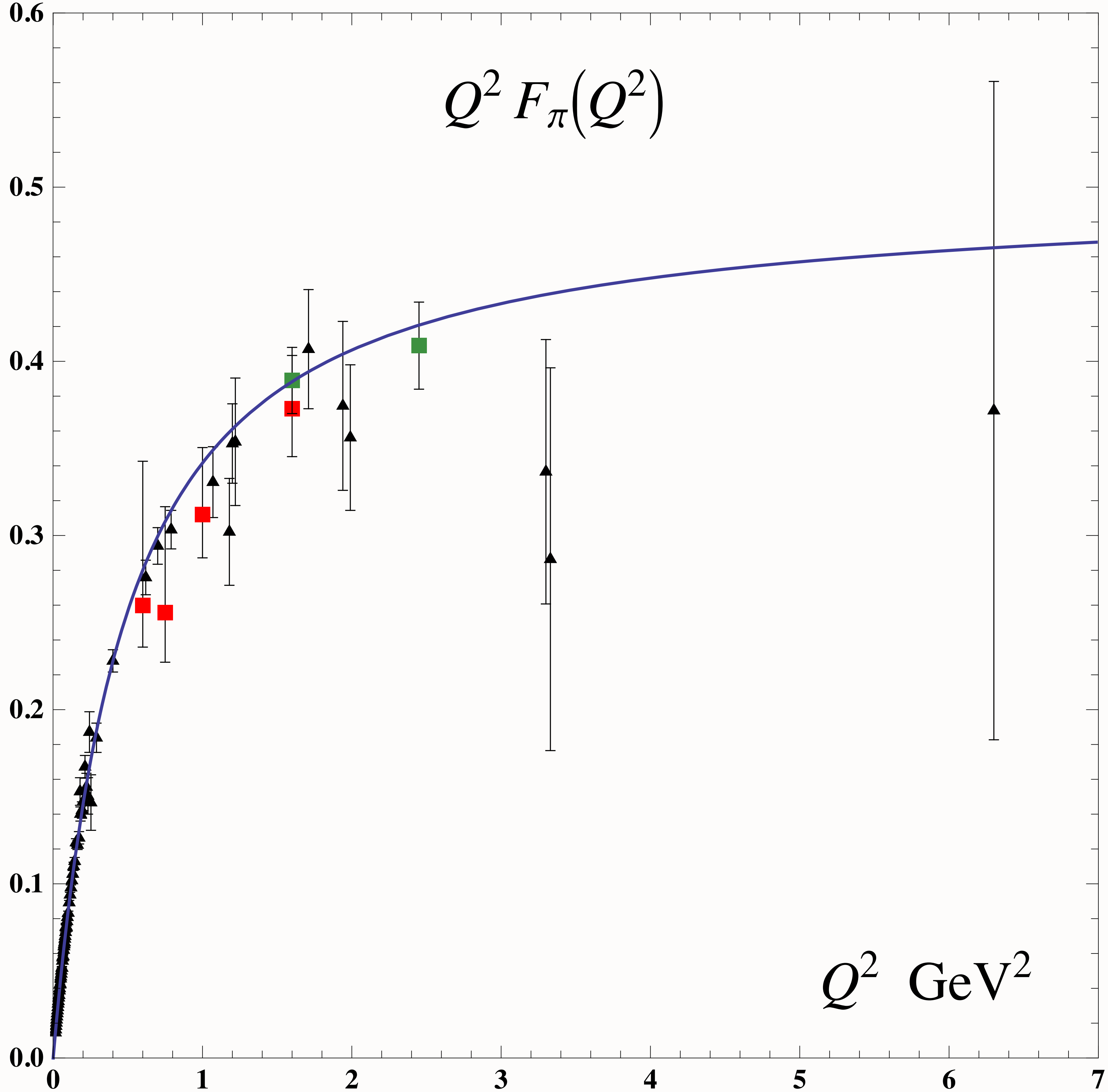
- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z).$$

- For large $Q^2 \gg 4\kappa^2$

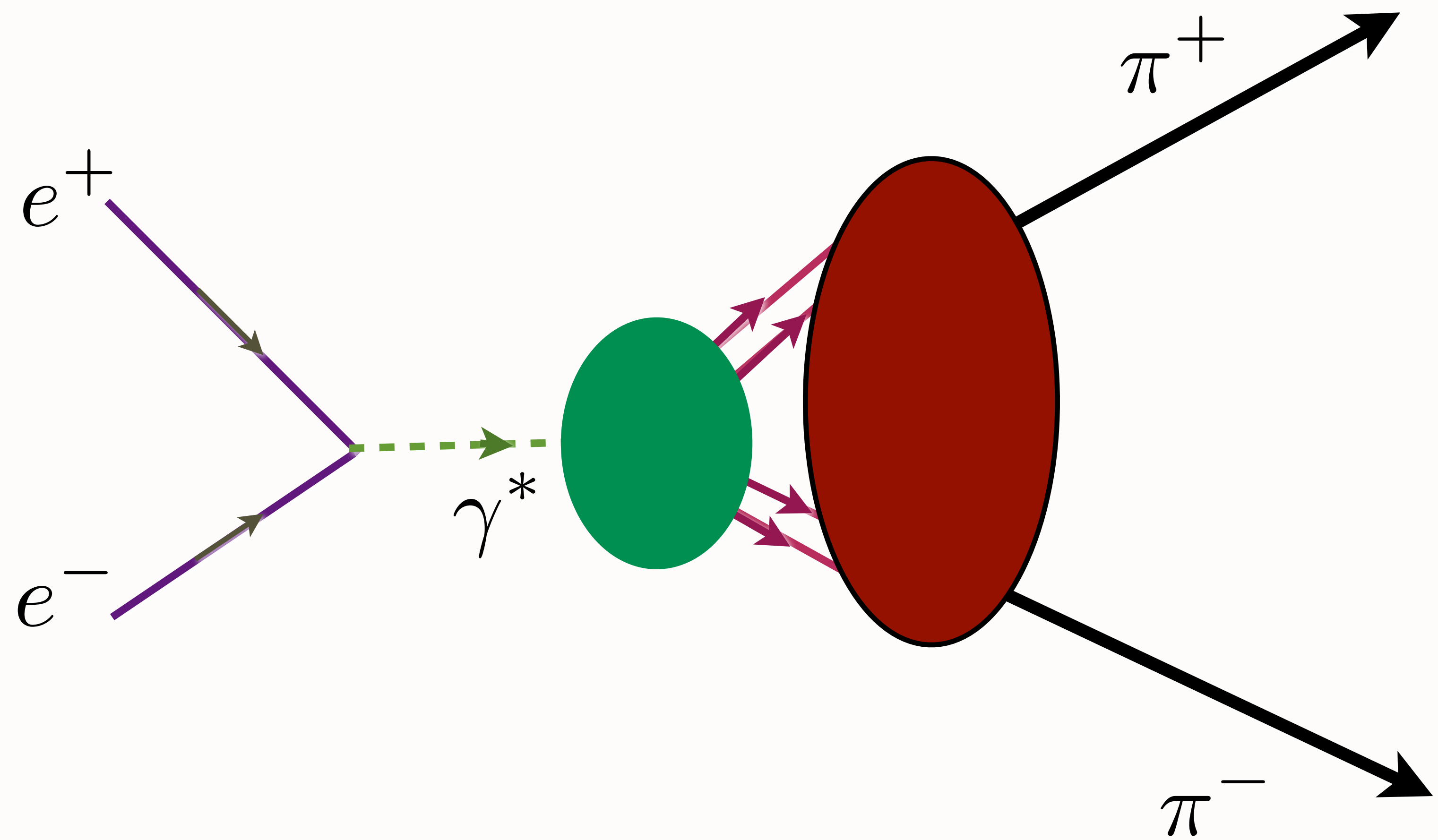
$$J_\kappa(Q, z) \to zQK_1(zQ) = J(Q, z),$$

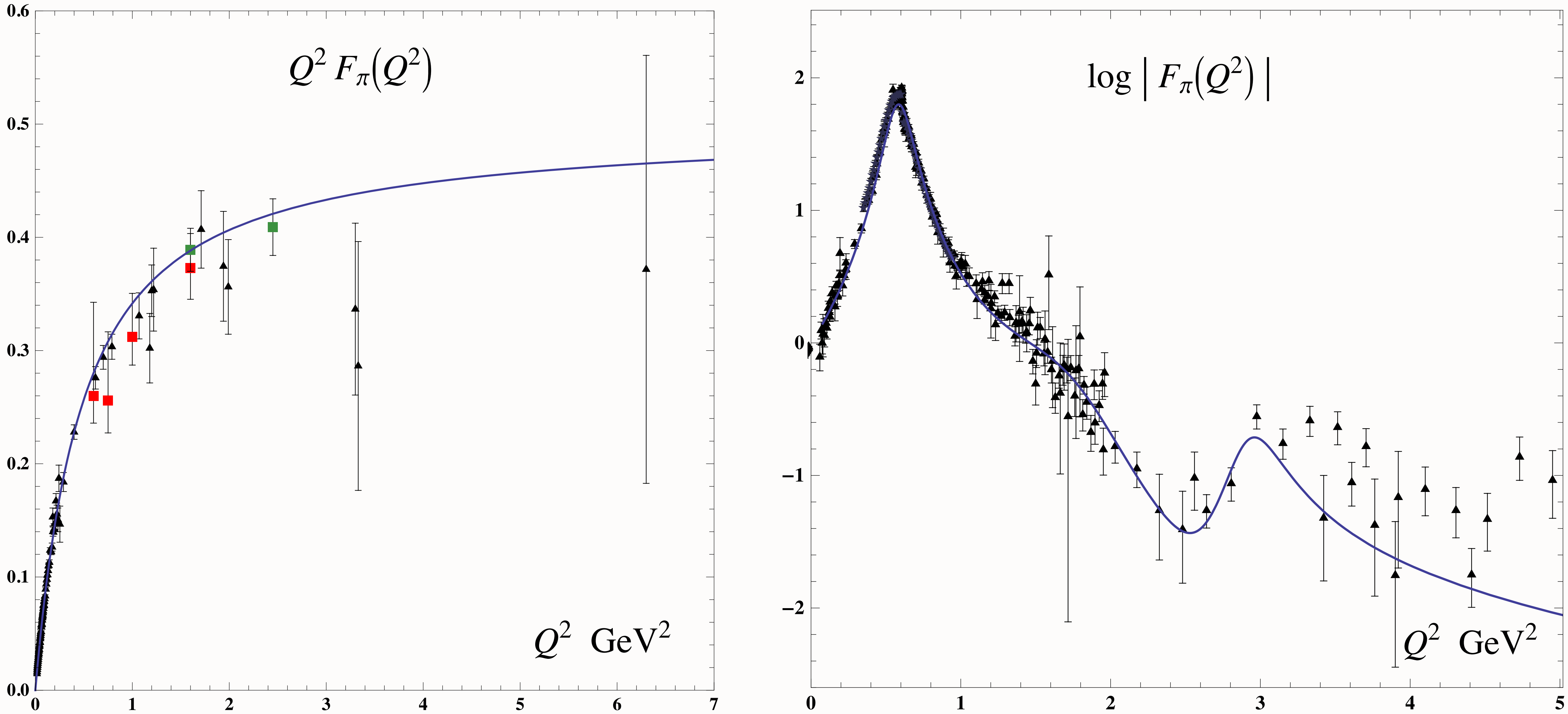
the external current decouples from the dilaton field.



$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
7

Dressed soft-wall current brings in higher Fock states and more vector meson poles





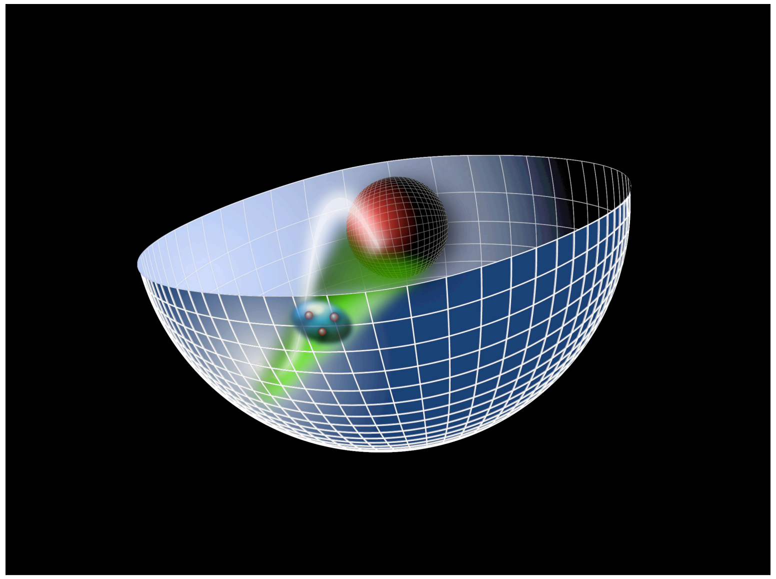
Spacelike and Timelike Pion Form Factor

Structure of the space- and time-like pion form factor in light-front holography for a truncation of the pion wave function up to twist four. Triangles are the data compilation from Baldini *et al.*, [42] red squares are JLAB 1 [43] and green squares are JLAB 2. [44]

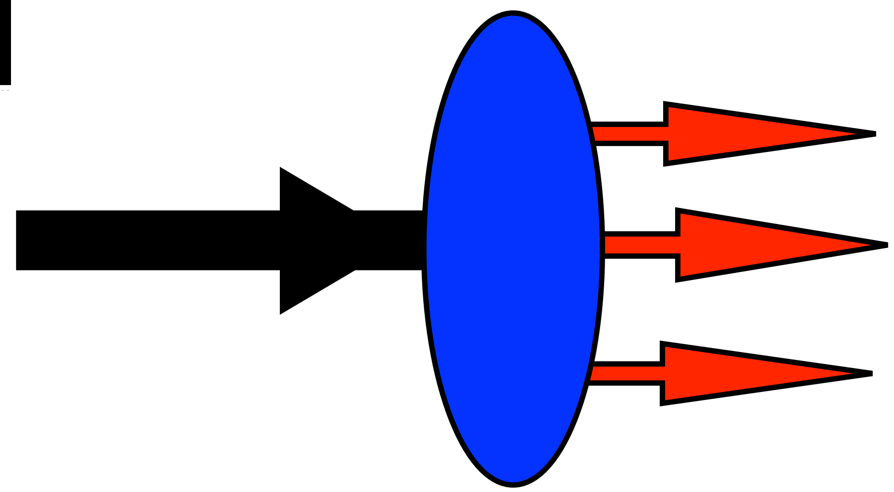
$$|\pi>= \psi_{\bar{q}q/\pi}|\bar{q}q> + \psi_{\bar{q}q\bar{q}q/\pi}|q\bar{q}\bar{q}q>$$

AdS/QCD $\kappa = 0.54$ GeV

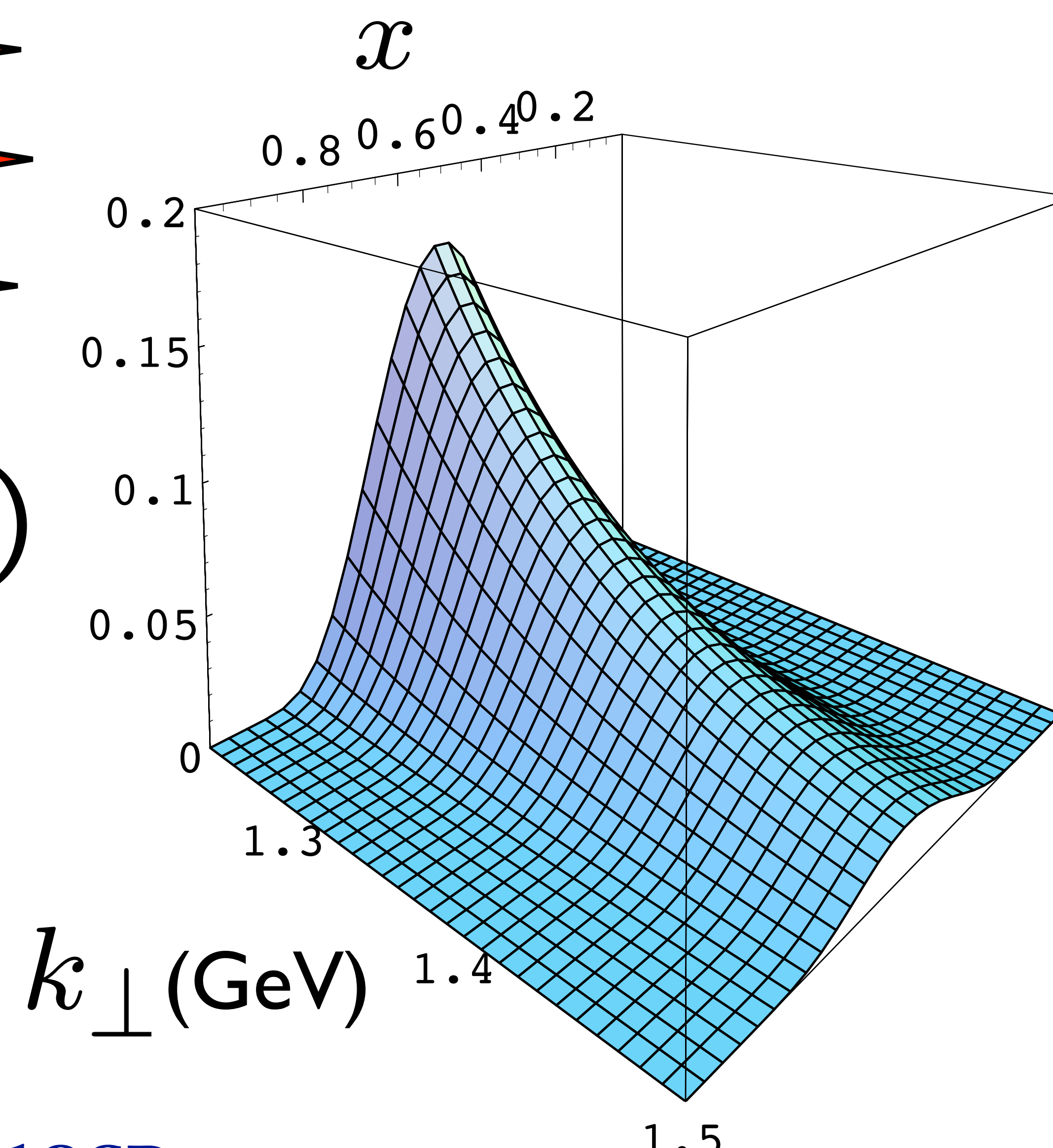
$\phi(z)$



- *Light-Front Holography*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

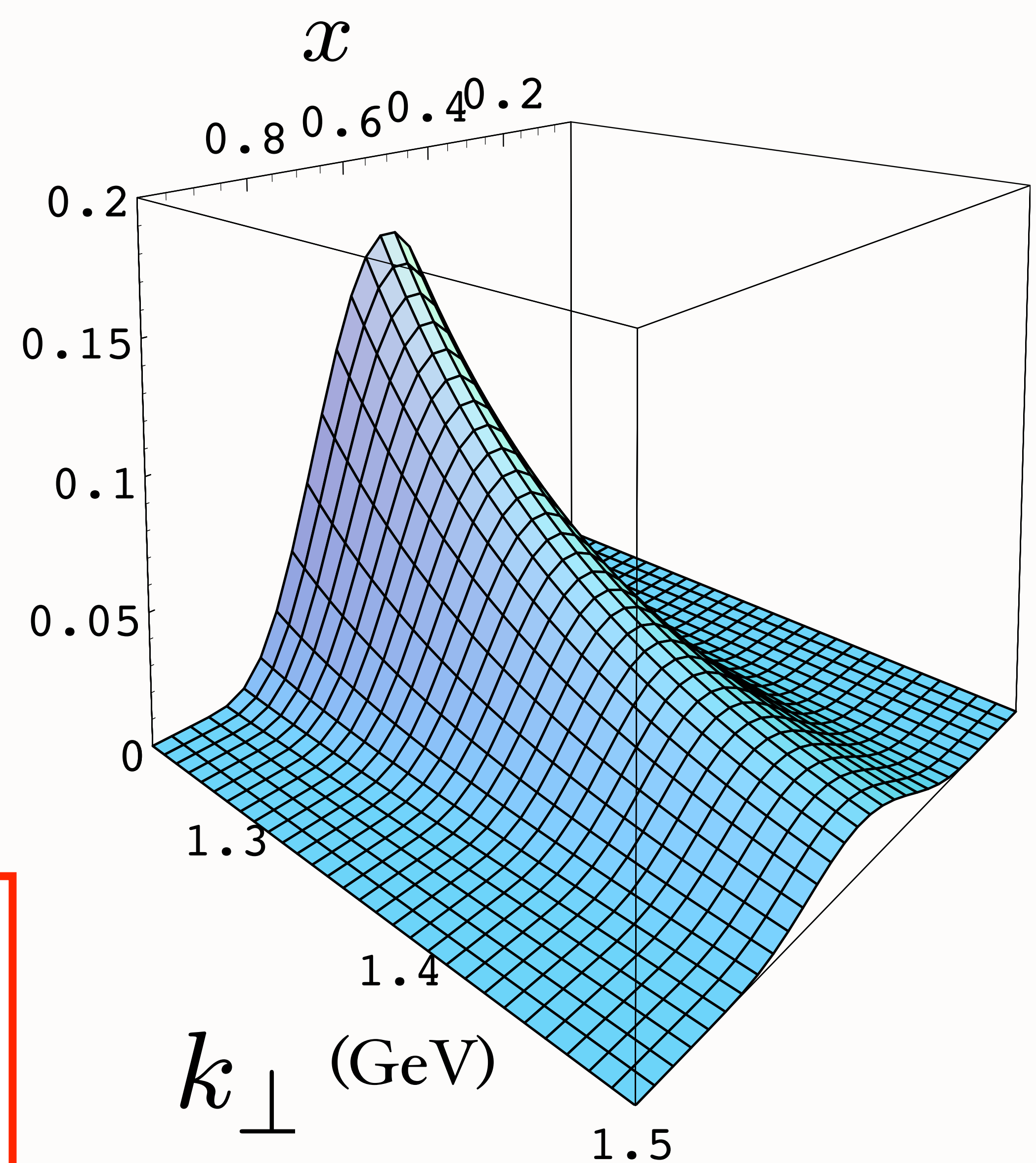


- *Light Front Wavefunctions:*

  Schrödinger Wavefunctions
  of Hadron Physics

Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb



$\psi_M(x, k_\perp^2)$

"Soft Wall" model

$\kappa = 0.375$ GeV

massless quarks

Note coupling

$k_\perp^2, x$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\boxed{\phi_M(x, Q_0) \propto \sqrt{x(1-x)}}$$

Connection of Confinement to TMDs

Second Moment of Pion Distribution Amplitude

$$< \xi^2 >= \int_{-1}^{1} d\xi \; \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

$$< \xi^2 >_\pi = 1/5 = 0.20 \qquad \phi_{asympt} \propto x(1-x)$$

$$< \xi^2 >_\pi = 1/4 = 0.25 \qquad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

Lattice (I) $< \xi^2 >_\pi = 0.28 \pm 0.03$ Donnellan et al.

Lattice (II) $< \xi^2 >_\pi = 0.269 \pm 0.039$ Braun et al.

Generalized parton distributions in AdS/QCD

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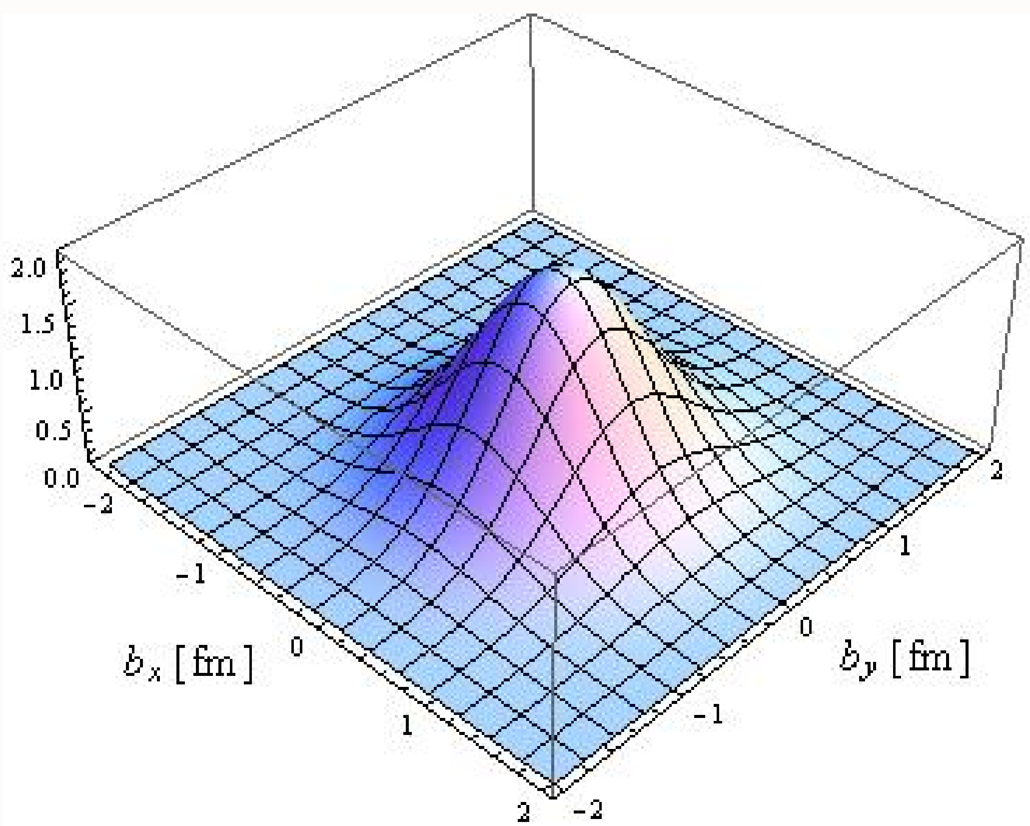
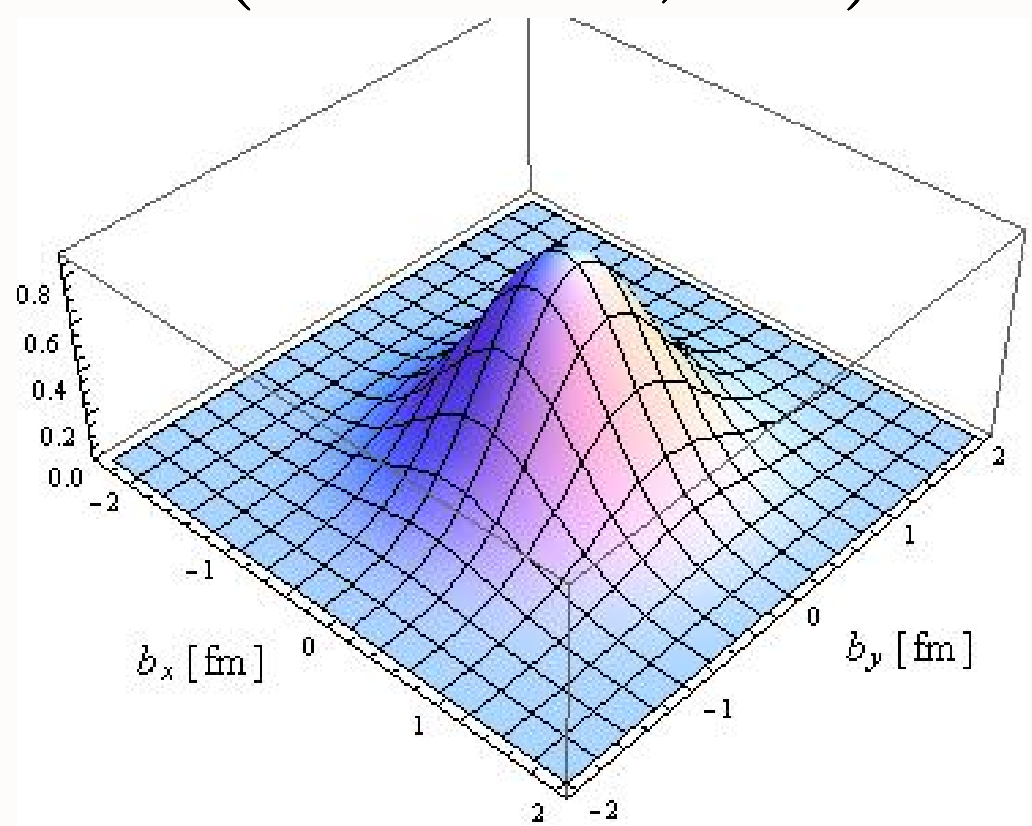
Kepler Center for Astro and Particle Physics,
Auf der Morgenstelle 14, D–72076 Tübingen, Germany

(Dated: January 19, 2011)

$$u(x = 0.1, \vec{b}_\perp)$$



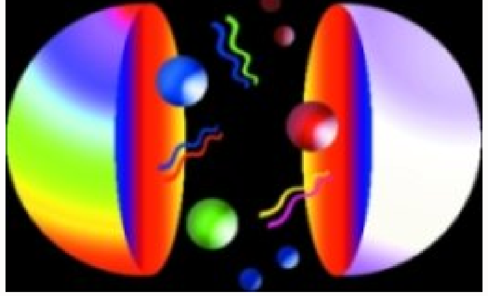
$$d(x = 0.1, \vec{b}_\perp)$$



Fermionic Modes and Baryon Spectrum

GdT and sjb, PRL 94, 201601 (2005)

Yukawa interaction in 5 dimensions



From Nick Evans

- Action for Dirac field in AdS$_{d+1}$ in presence of dilaton background $\varphi(z)$ [Abidin and Carlson (2009)]

$$S = \int d^{d+1} \sqrt{g} e^{\varphi}(z) \left(i \overline{\Psi} e^M_A \Gamma^A D_M \Psi + h.c + \varphi(z) \overline{\Psi} \Psi - \mu \overline{\Psi} \Psi \right)$$

$$\phi(z) = e^{\kappa^2 z^2}$$

- Factor out plane waves along 3+1: $\Psi_P(x^\mu, z) = e^{-iP\cdot x} \Psi(z)$

$$\left[i \left(z \eta^{\ell m} \Gamma_\ell \partial_m + 2\Gamma_z \right) + \mu R + \kappa^2 z \right] \Psi(x^\ell) = 0.$$

- Solution $\left(\nu = \mu R - \frac{1}{2},\ \nu = L + 1\right)$

$$\Psi_+(z) \sim z^{\frac{5}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^\nu(\kappa^2 z^2), \quad \Psi_-(z) \sim z^{\frac{7}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^{\nu+1}(\kappa^2 z^2)$$

- Eigenvalues (how to fix the overall energy scale, see arXiv:1001.5193)

$$\mathcal{M}^2 = 4\kappa^2(n + L + 1)$$

positive parity

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ by shifting dimensions
- Large N_C: $\mathcal{M}^2 = 4\kappa^2(N_C + n + L - 2) \implies \mathcal{M} \sim \sqrt{N_C}\, \Lambda_{\mathrm{QCD}}$

Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0,$$

in terms of the matrix-valued operator Π

$$\nu = L + 1$$

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right),$$

and its adjoint $\Pi^\dagger$, with commutation relations

Soft Wall

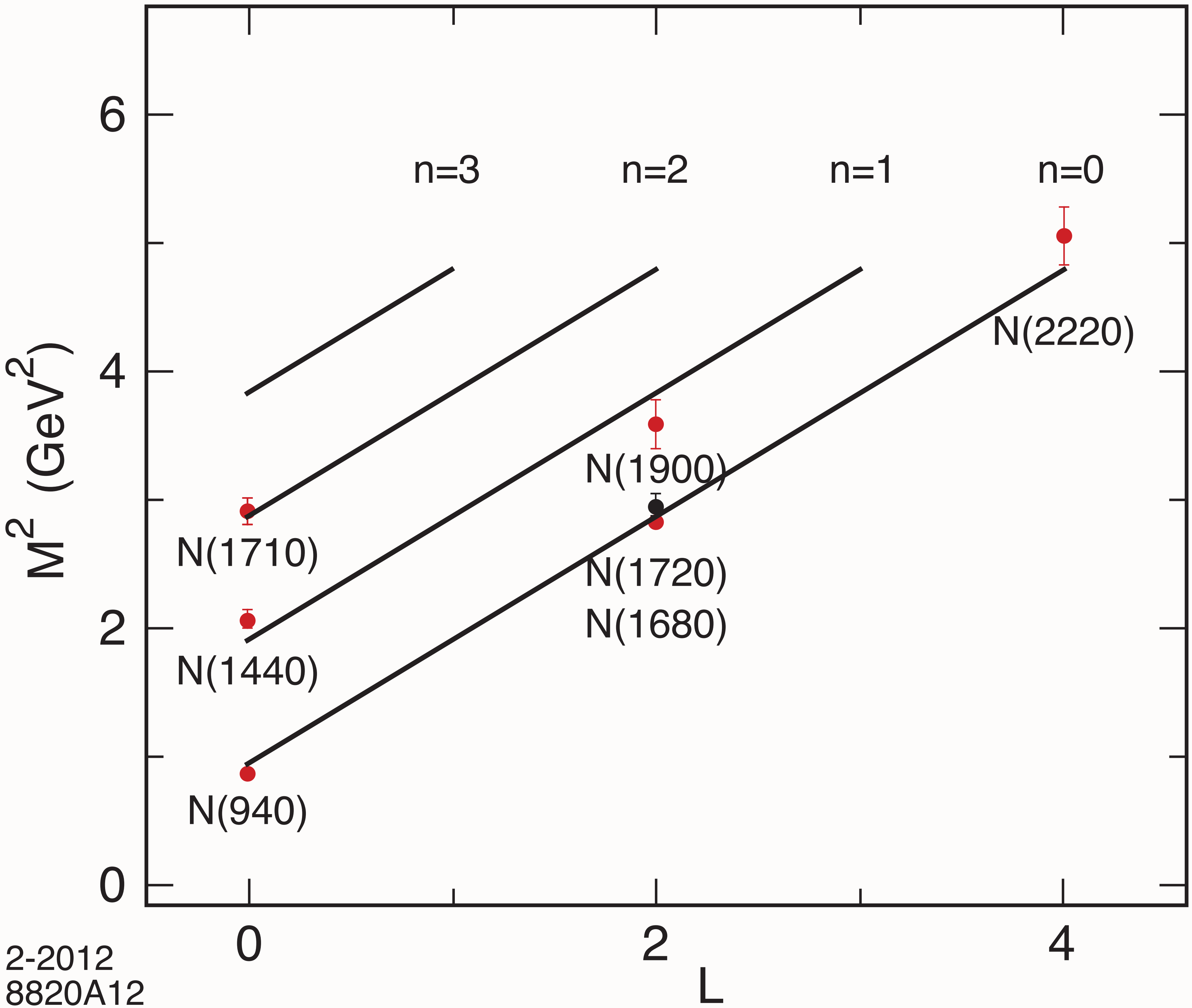
$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \left(\frac{2\nu+1}{\zeta^2} - 2\kappa^2\right)\gamma_5.$$

- Solutions to the Dirac equation

$$\begin{aligned}\psi_+(\zeta) &\sim z^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2),\\ \psi_-(\zeta) &\sim z^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2).\end{aligned}$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1).$$



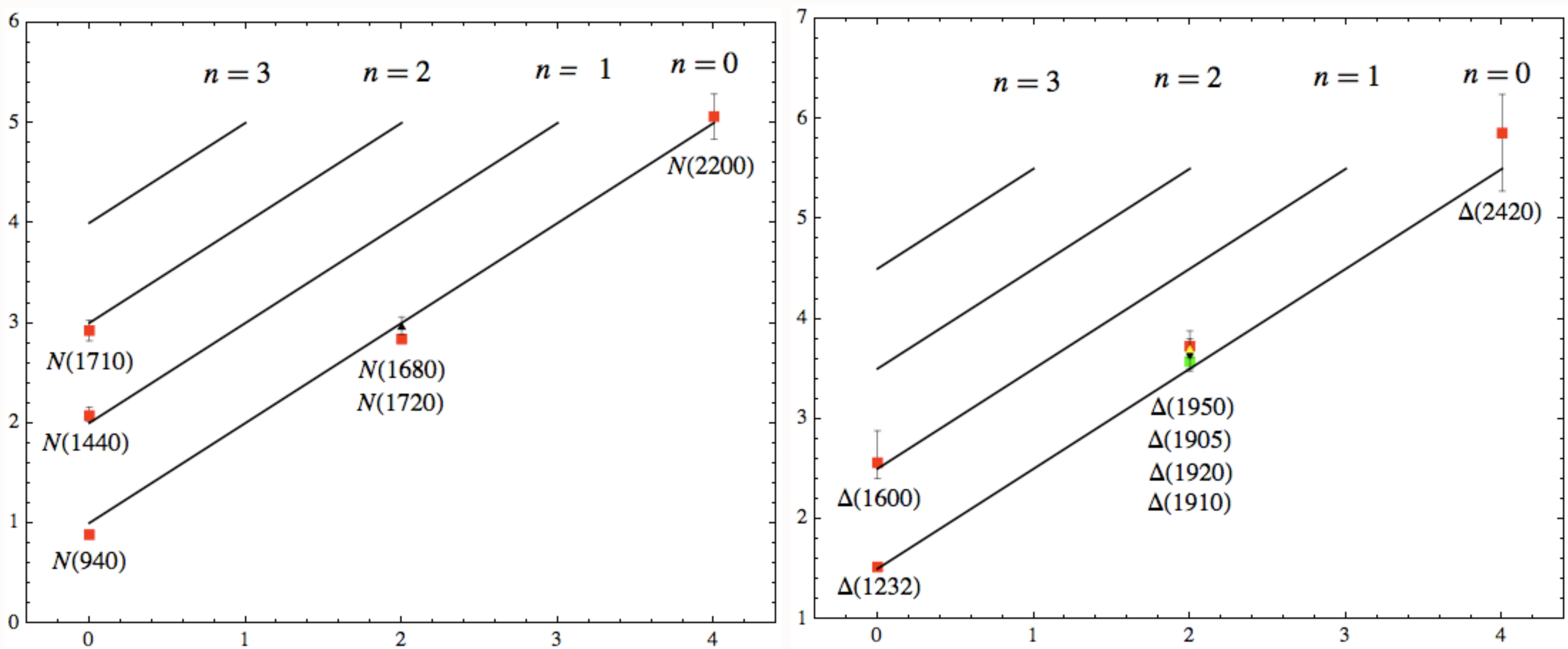
6
4
2
0
M^2 (GeV^2)
n=3
n=2
n=1
n=0
N(2220)
N(1900)
N(1710)
N(1720)
N(1680)
N(1440)
N(940)
0
2
4
L
2-2012
8820A12

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

- **Δ spectrum identical to Forkel and Klempt, Phys. Lett. B 679, 77 (2009)**

Same multiplicity of states for mesons and baryons!

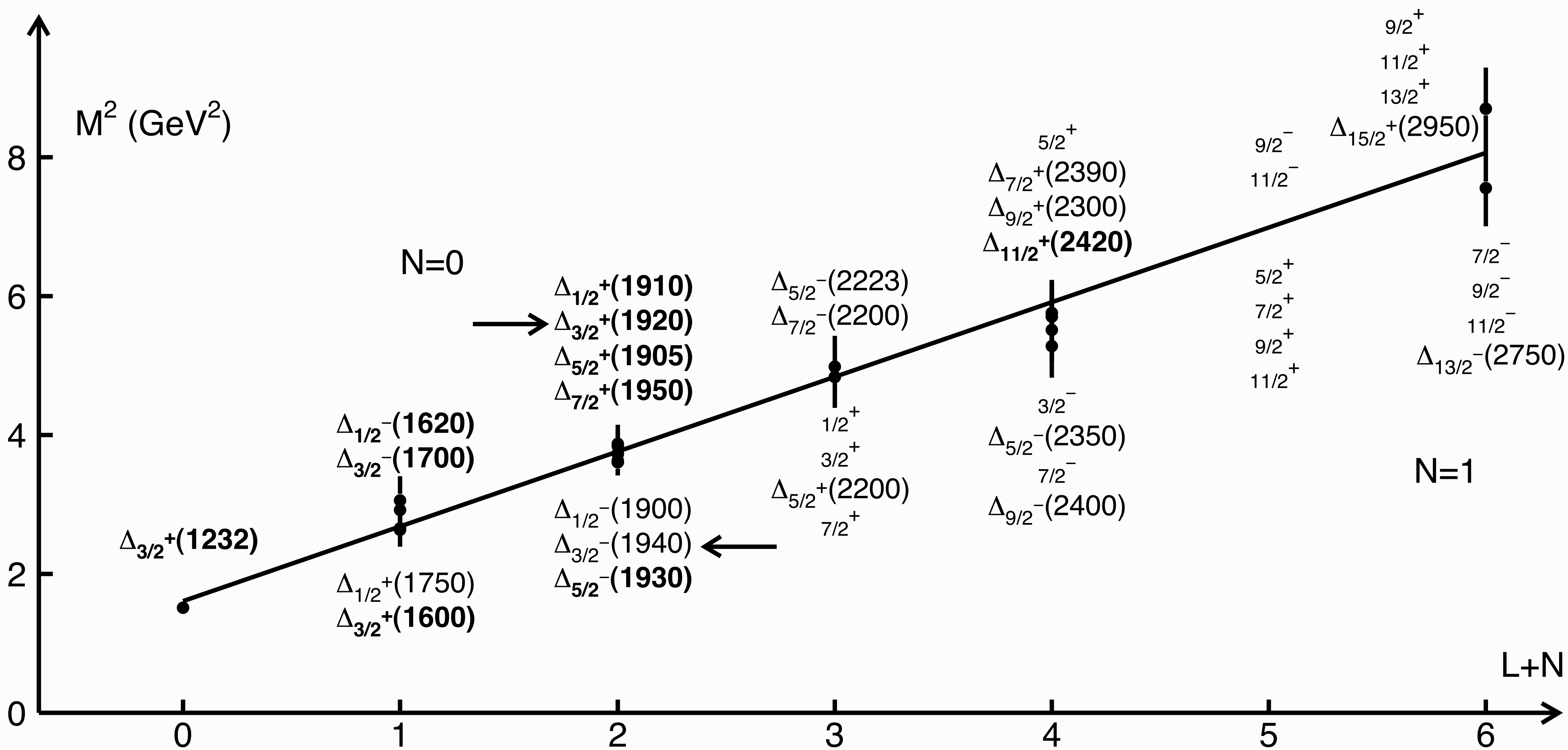
$\mathcal{M}^2$



L

Parent and daughter **56** Regge trajectories for the N and Δ baryon families for $\kappa = 0.5$ GeV

- **Δ spectrum identical to Forkel and Klempt, Phys. Lett. B 679, 77 (2009)**



E. Klempt *et al.*: Δ^* resonances, quark models, chiral symmetry and AdS/QCD

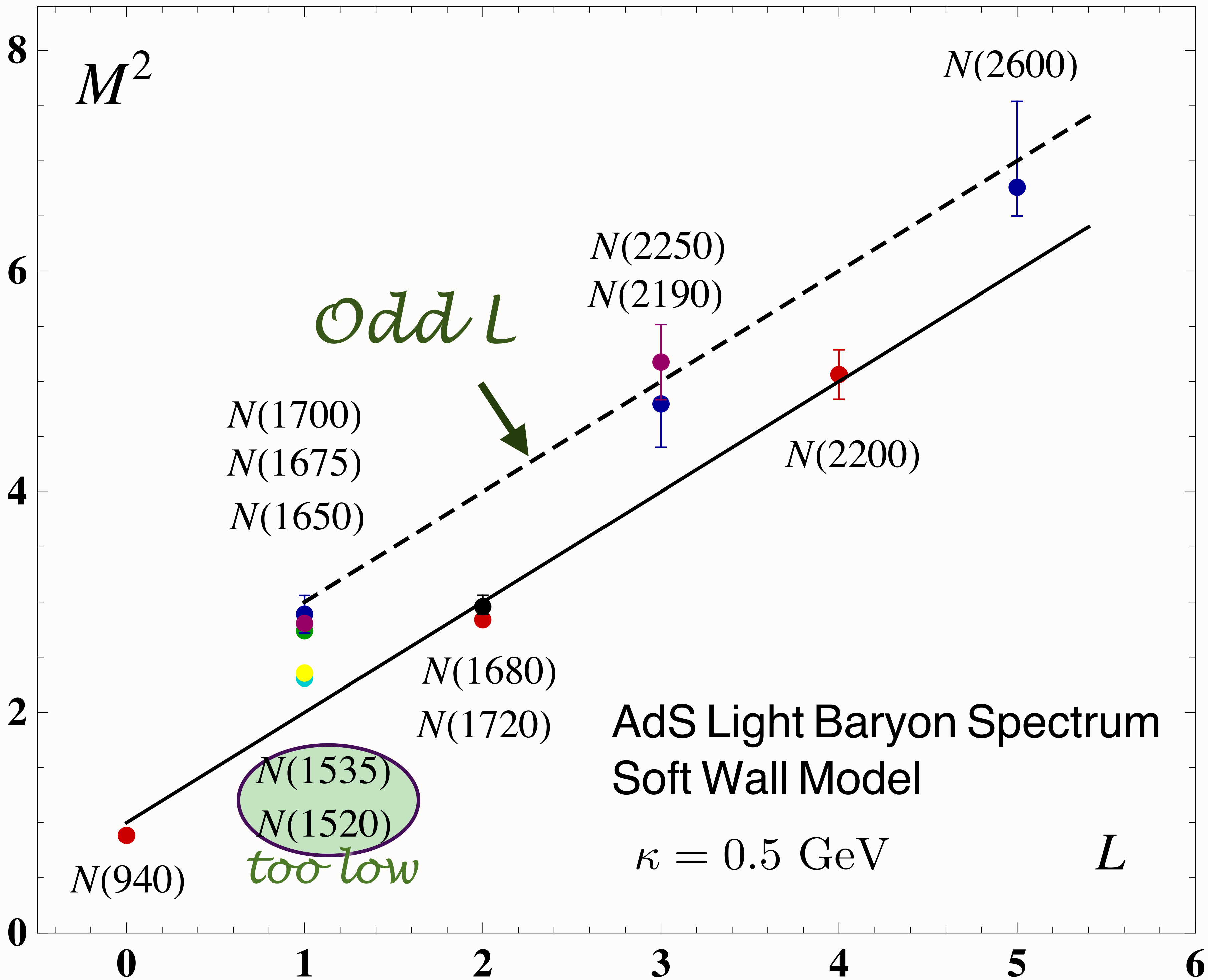
H. Forkel, M. Beyer and T. Frederico, JHEP **0707** (2007) 077.
H. Forkel, M. Beyer and T. Frederico, Int. J. Mod. Phys. E **16** (2007) 2794.

Positive Parity Nucleons

Negative Parity Nucleons

$$M^2 = 4\kappa^2(n + L + 1)$$

$$M^2 = 4\kappa^2(n + L + 2)$$



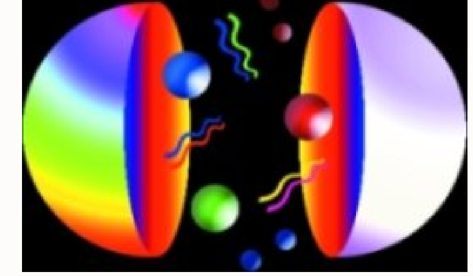
$L + 1 = \nu = \mu R - 1/2$ **(even P)**

$L + 1 = \nu = \mu R + 1/2$ **(odd P)**

Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int d\zeta\, \psi_+^2(\zeta) = \int d\zeta\, \psi_-^2(\zeta)$$

- Eigenvalues

$$\mathcal{M}^2_{n,L,S=1/2} = 4\kappa^2\left(n+L+1\right)$$

- “Chiral partners”

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta \, J(Q,\zeta) |\psi_-(\zeta)|^2,$$

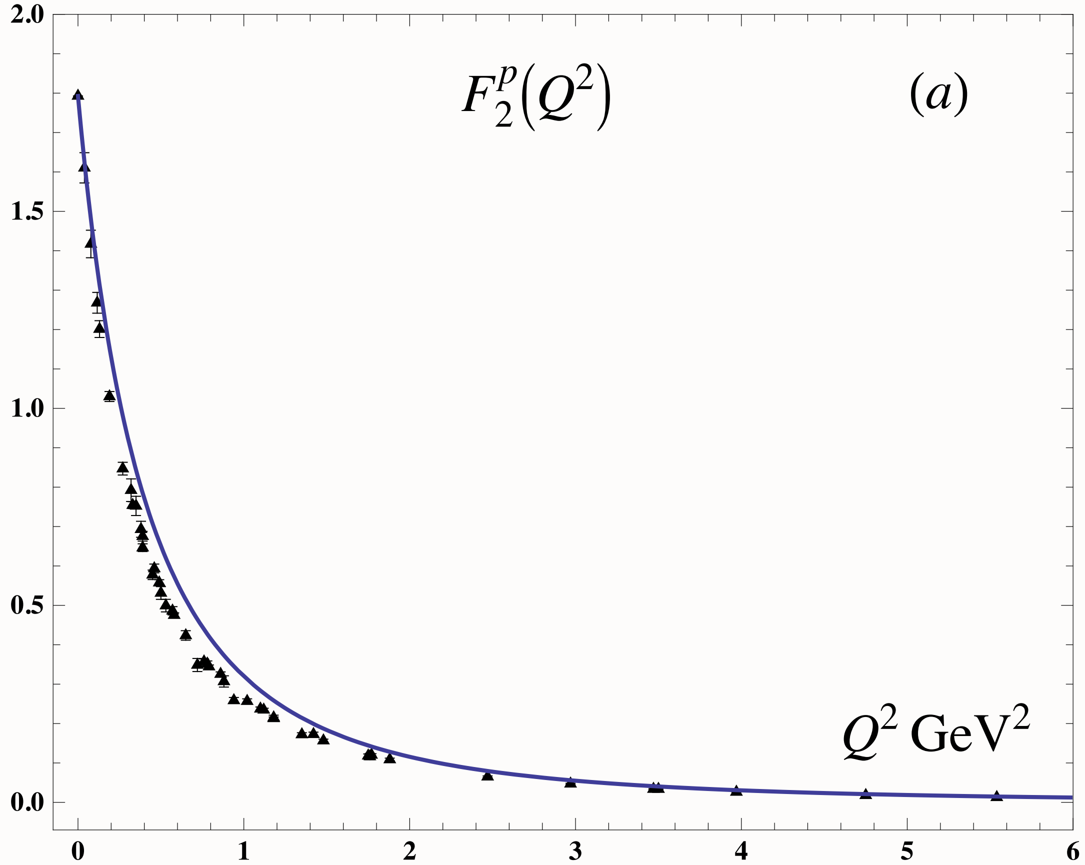
where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

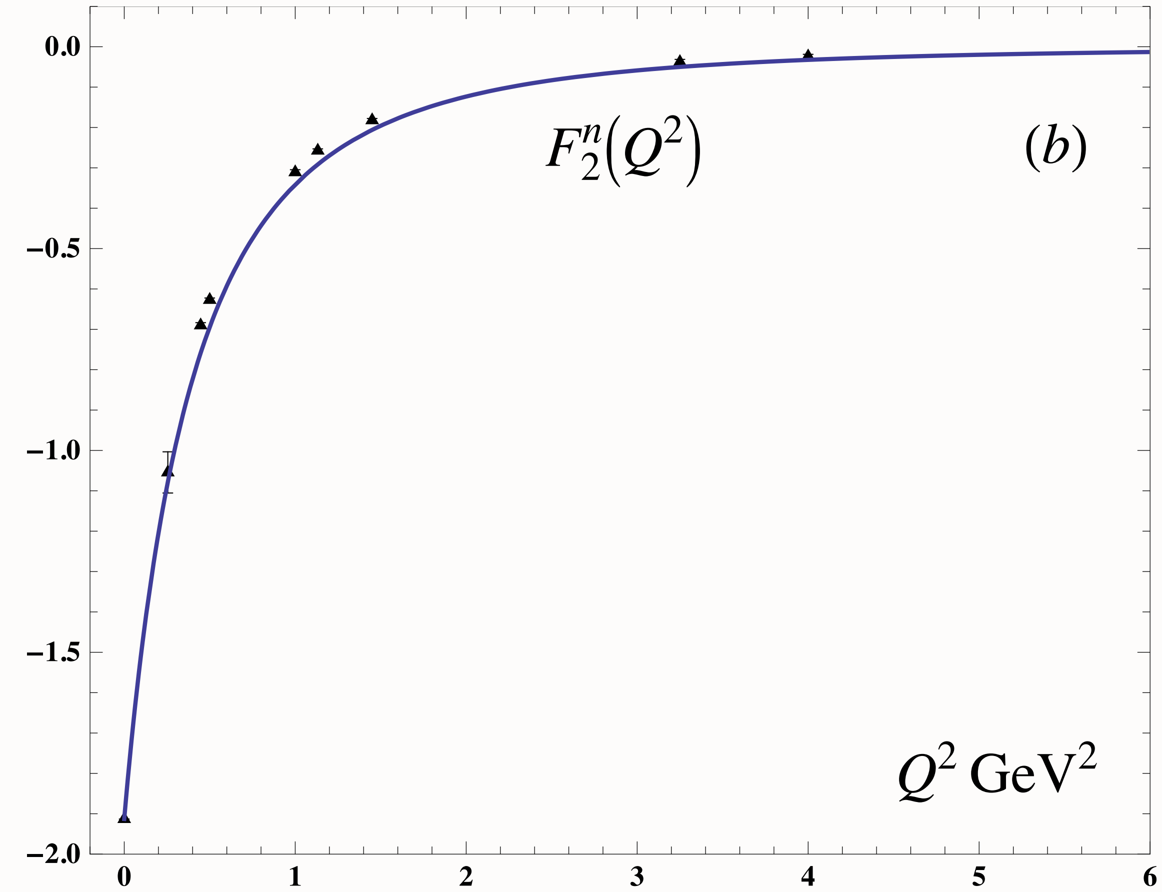
- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta \, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],$$

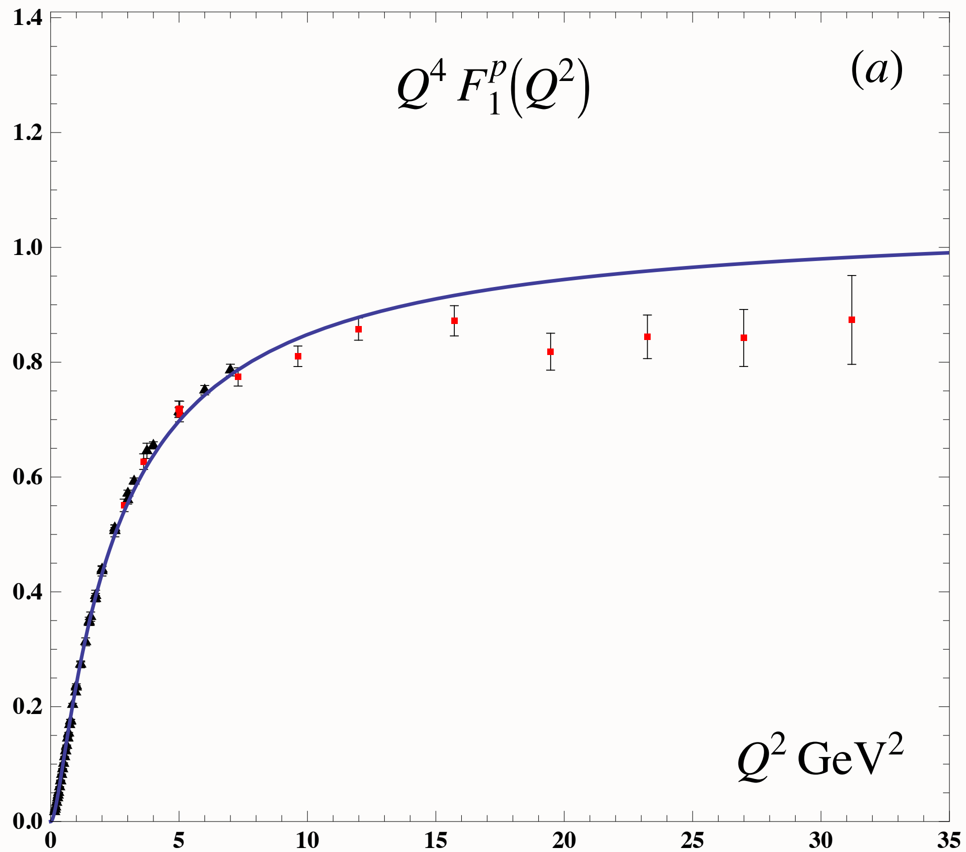
where $F_1^p(0) = 1, \; F_1^n(0) = 0$.



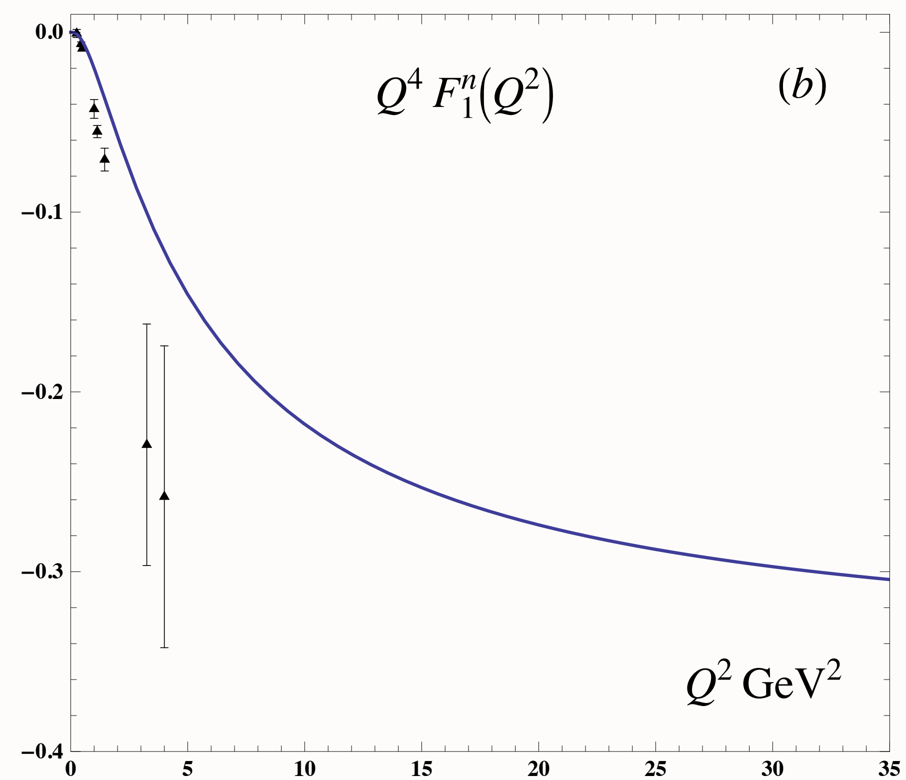
$F_2^p(Q^2)$
(a)
Q^2 GeV^2



$F_2^n(Q^2)$
(b)
Q^2 GeV^2



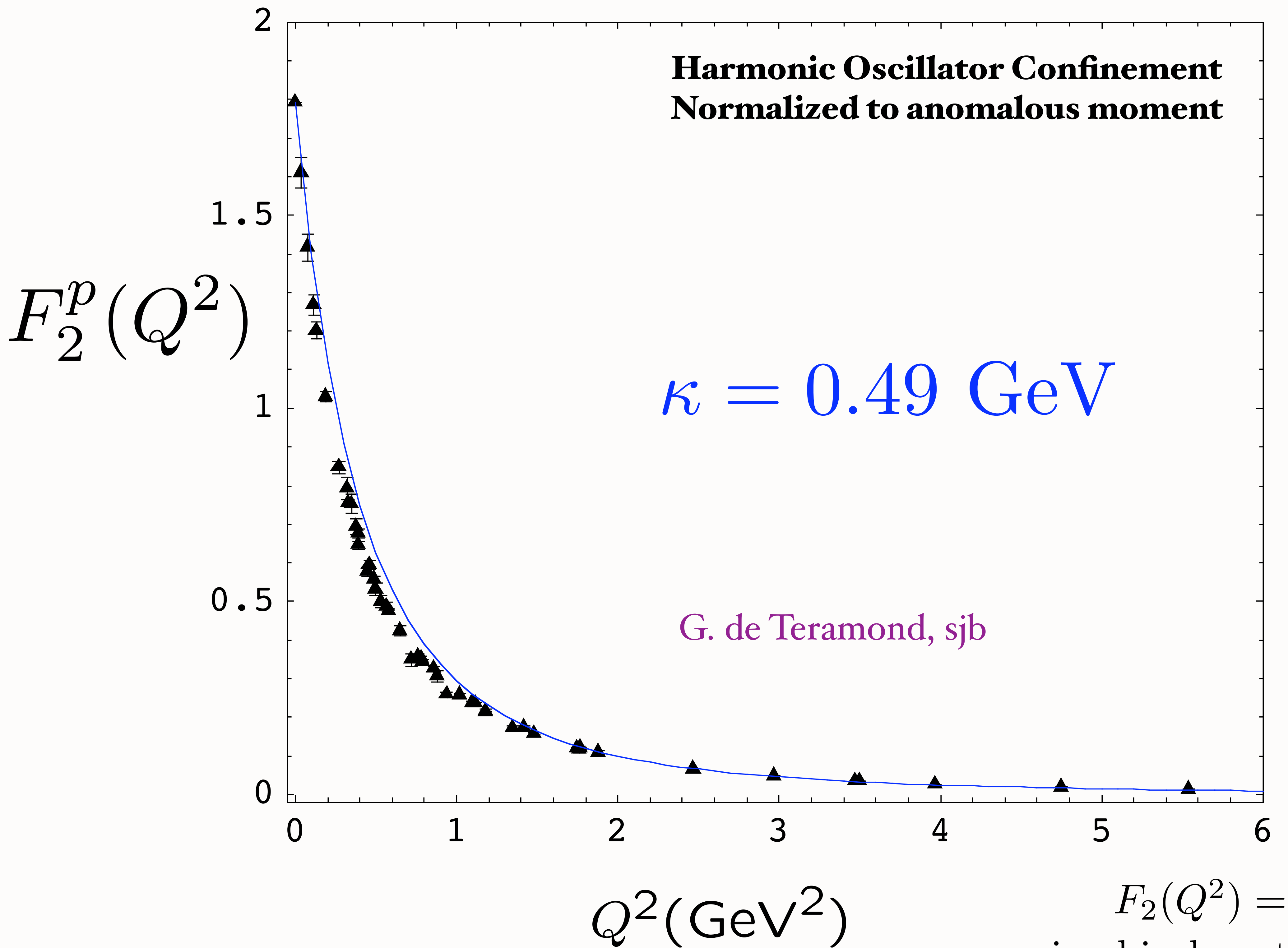
$Q^4 F_1^p(Q^2)$
(a)
Q^2 GeV^2



$Q^4 F_1^n(Q^2)$
(b)
Q^2 GeV^2

Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs



AdS/QCD No chiral divergence!

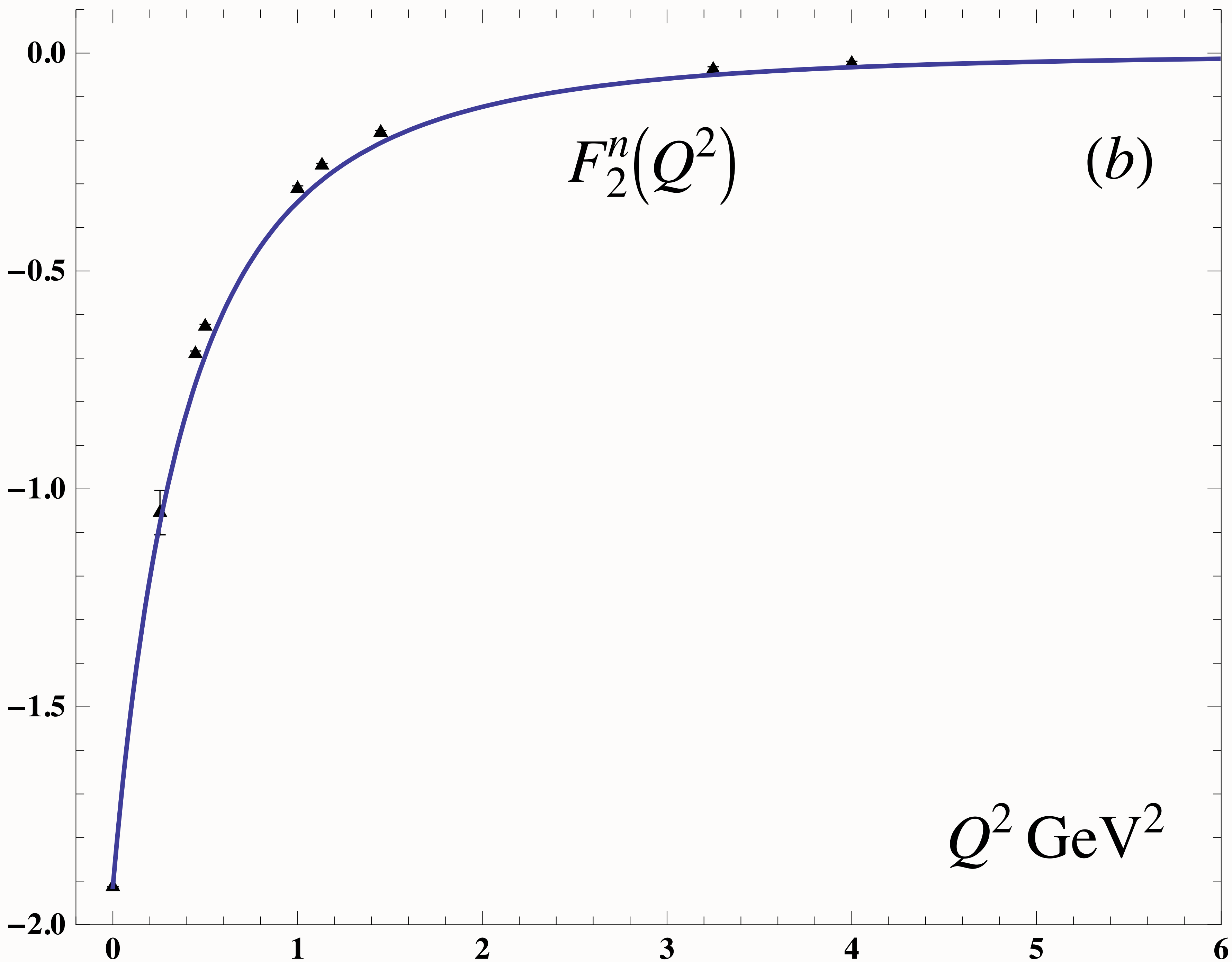
$$F_2(Q^2) = 1 + \mathcal{O}\frac{Q^2}{m_\pi m_p}$$

in chiral perturbation theory

Spacelike Neutron Pauli Form Factor

Preliminary

From overlap of $L = 1$ and $L = 0$ LFWFs



Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \to N^*(1440)$: $\quad \Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$

- Transition form factor

$$F_{1\,N\to N^*}^{\,p}(Q^2) = R^4 \int \frac{dz}{z^4}\, \Psi_+^{n=1,L=0}(z) V(Q,z) \Psi_+^{n=0,L=0}(z)$$

- Orthonormality of Laguerre functions $\quad \left(F_{1\,N\to N^*}^{\,p}(0) = 0, \quad V(Q=0,z)=1\right)$

$$R^4 \int \frac{dz}{z^4}\, \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

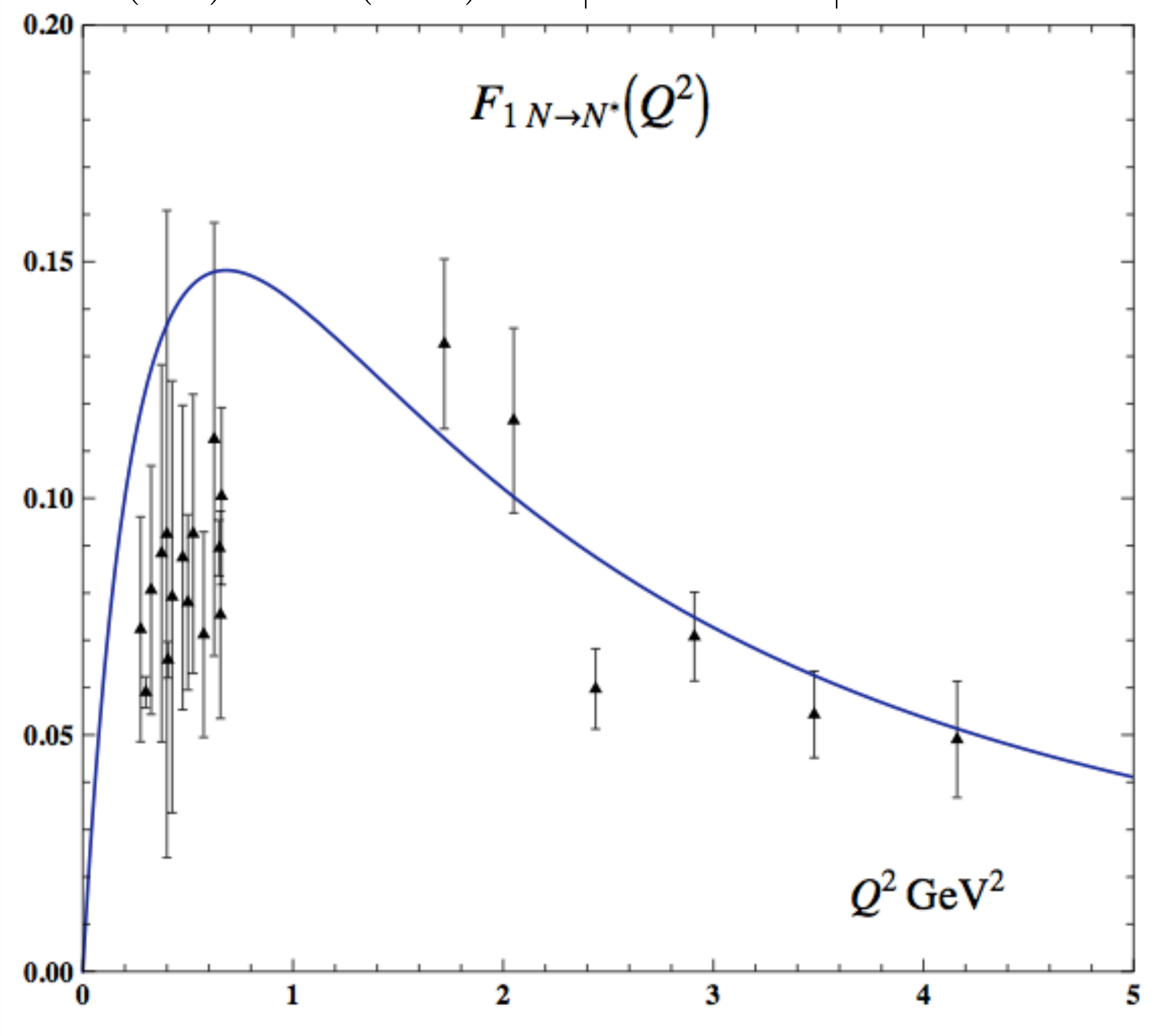
$$F_{1\,N\to N^*}^{\,p}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

with ${\mathcal{M}_{\rho_n}}^2 \to 4\kappa^2(n+1/2)$

de Teramond, sjb

Consistent with counting rule, twist 3

$N(940) \to N^*(1440)$: $\Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$



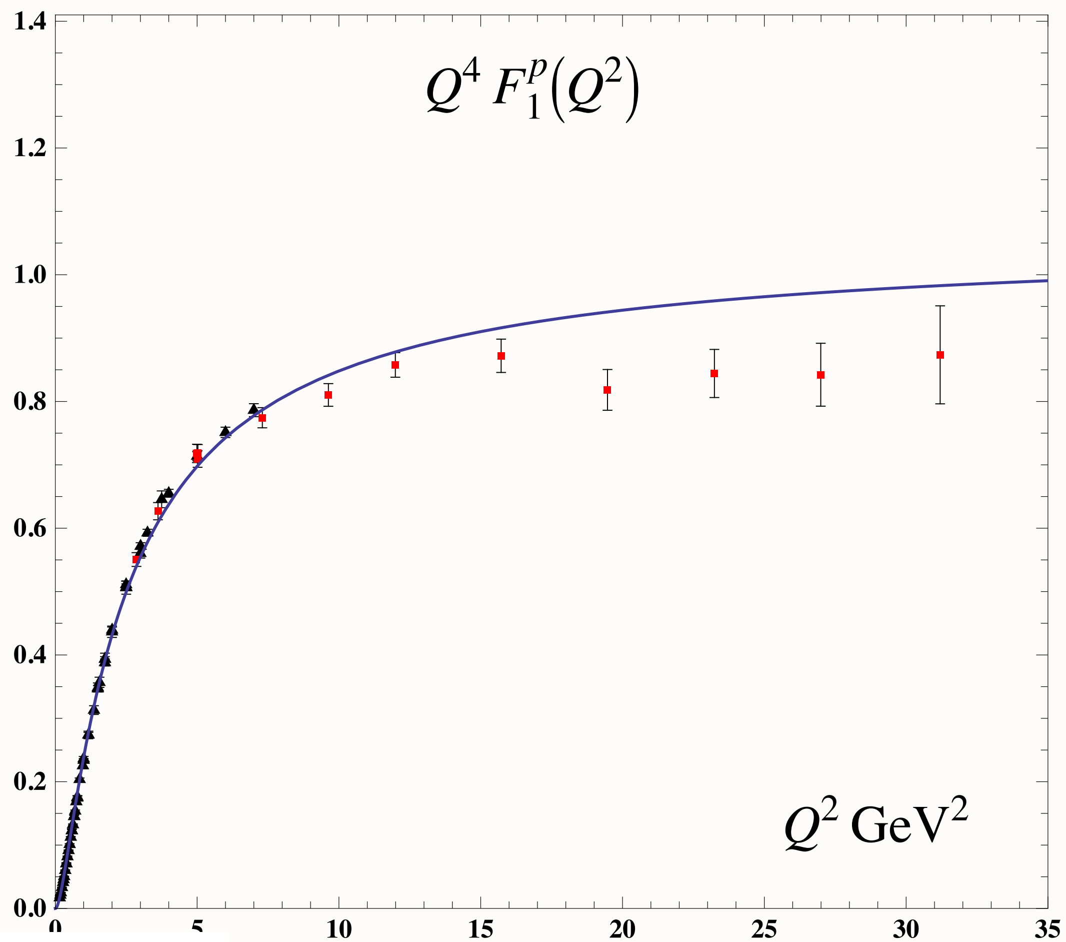
Data from I. Aznauryan, *et al.* CLAS (2009)

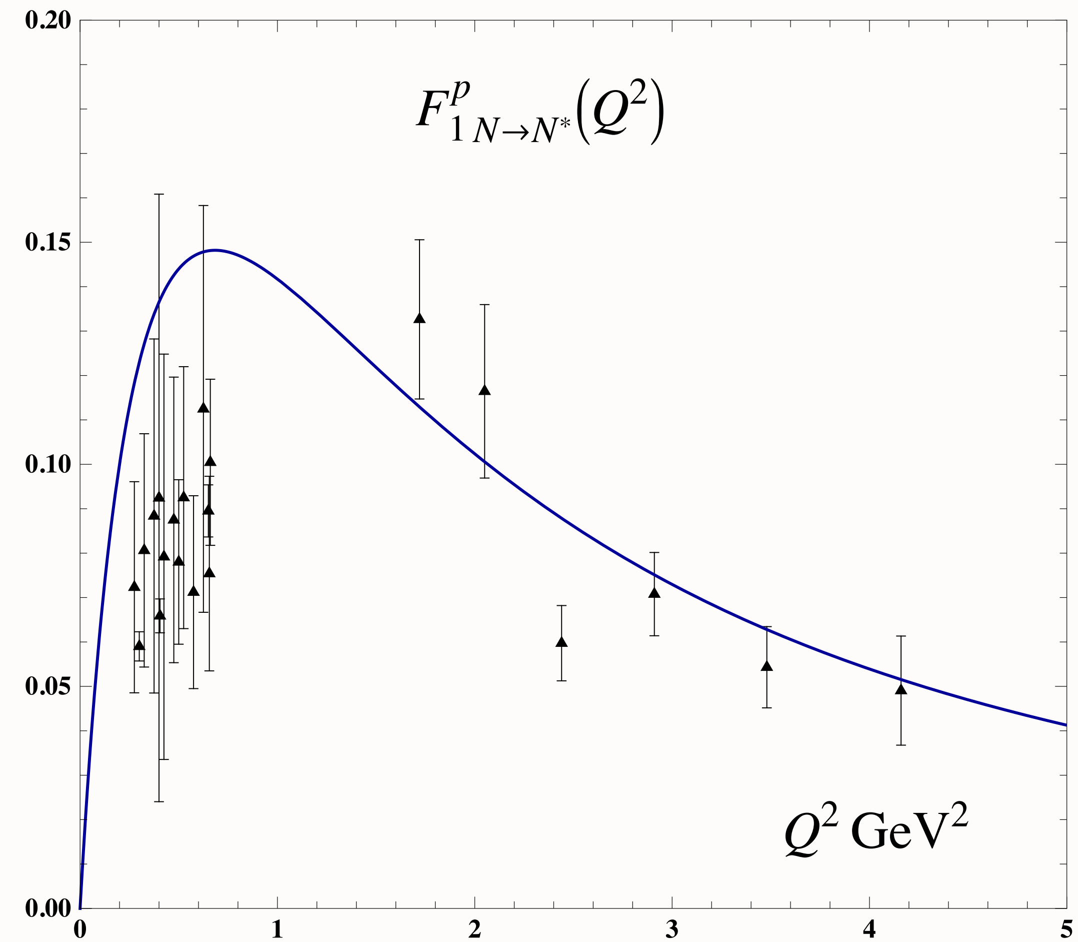
$$\boxed{F_1{}^p_{N\to N^*}(Q^2) = \frac{2\sqrt{2}}{3}\,\frac{\frac{Q^2}{M_P^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}}$$

with $\mathcal{M}_{\rho_n}{}^2 \to 4\kappa^2(n+1/2)$

Nucleon Elastic and Transition Form Factors

$$F_1^p(Q^2) = \frac{1}{\left(1+\frac{Q^2}{M_\rho^2}\right)\left(1+\frac{Q^2}{M_{\rho'}^2}\right)}, \qquad F^p_{1\,N\to N^*}(Q^2) = \frac{\sqrt{2}}{3}\frac{\frac{Q^2}{M_\rho^2}}{\left(1+\frac{Q^2}{M_\rho^2}\right)\left(1+\frac{Q^2}{M_{\rho'}^2}\right)\left(1+\frac{Q^2}{M_{\rho''}^2}\right)},$$





Dirac proton form factors in light-front holographic QCD. Left: scaling of proton elastic form factor $Q^4 F_1^p(Q^2)$. Right: proton transition form factor $F^p_{1\,N\to N^*}(Q^2)$ to the first radial excited state. Data compilation from Diehl [32] (left) and JLAB [33] (right).

Guy de Teramond, sjb

Form Factors in AdS/QCD

$$F(Q^2) = \frac{1}{1 + \frac{Q^2}{\mathcal{M}_\rho^2}}, \quad N = 2,$$

$$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}, \quad N = 3,$$

$$\cdots$$

$$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\cdots\left(1 + \frac{Q^2}{\mathcal{M}^2_{\rho^{N-2}}}\right)}, \quad N,$$

Positive Dilaton Background $\exp\left(+\kappa^2 z^2\right)$ $\quad \mathcal{M}_n^2 = 4\kappa^2\left(n + \frac{1}{2}\right)$

$$F(Q^2) \to (N-1)!\left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)} \qquad Q^2 \to \infty$$

Constituent Counting

Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**

- **Trivial LF vacuum.**

Proton spin
carried by quark angular momentum!

- **Massless Pion**

- **Hadron Eigenstates have LF Fock components of different L^z**

- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$

$$J^z = +1/2 :< L^z >= 1/2, < S_q^z = 0 >$$

- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**

- **Label State by minimum L as in Atomic Physics**

- **Minimum L dominates at short distances**

- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at z=0.**

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS$_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} \, \frac{1}{g_5^2} \, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta \, J_0(\zeta Q) \, \alpha_s^{AdS}(\zeta)$$

- Solution

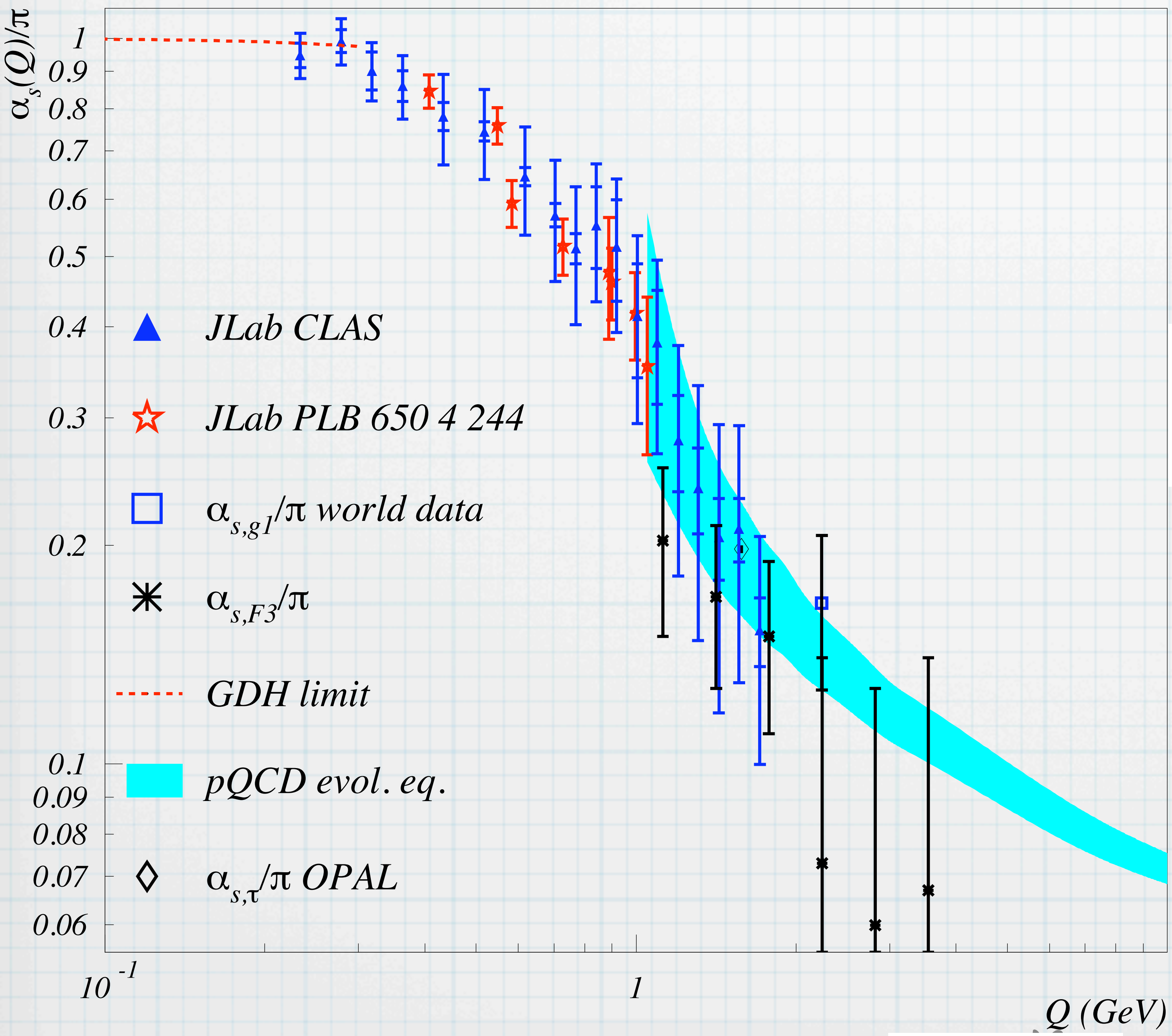
$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) \, e^{-Q^2/4\kappa^2}.$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Nearly conformal QCD?

Define α_s from Björkén sum,

$$\Gamma_1^{p-n} \equiv \int_0^1 dx \left(g_1^p(x,Q^2) - g_1^n(x,Q^2)\right) = \frac{1}{6} g_A \left(1 - \frac{\alpha_{s,g_1}}{\pi}\right)$$



Deur, de Teramond, sjb

g1 = spin dependent structure functio

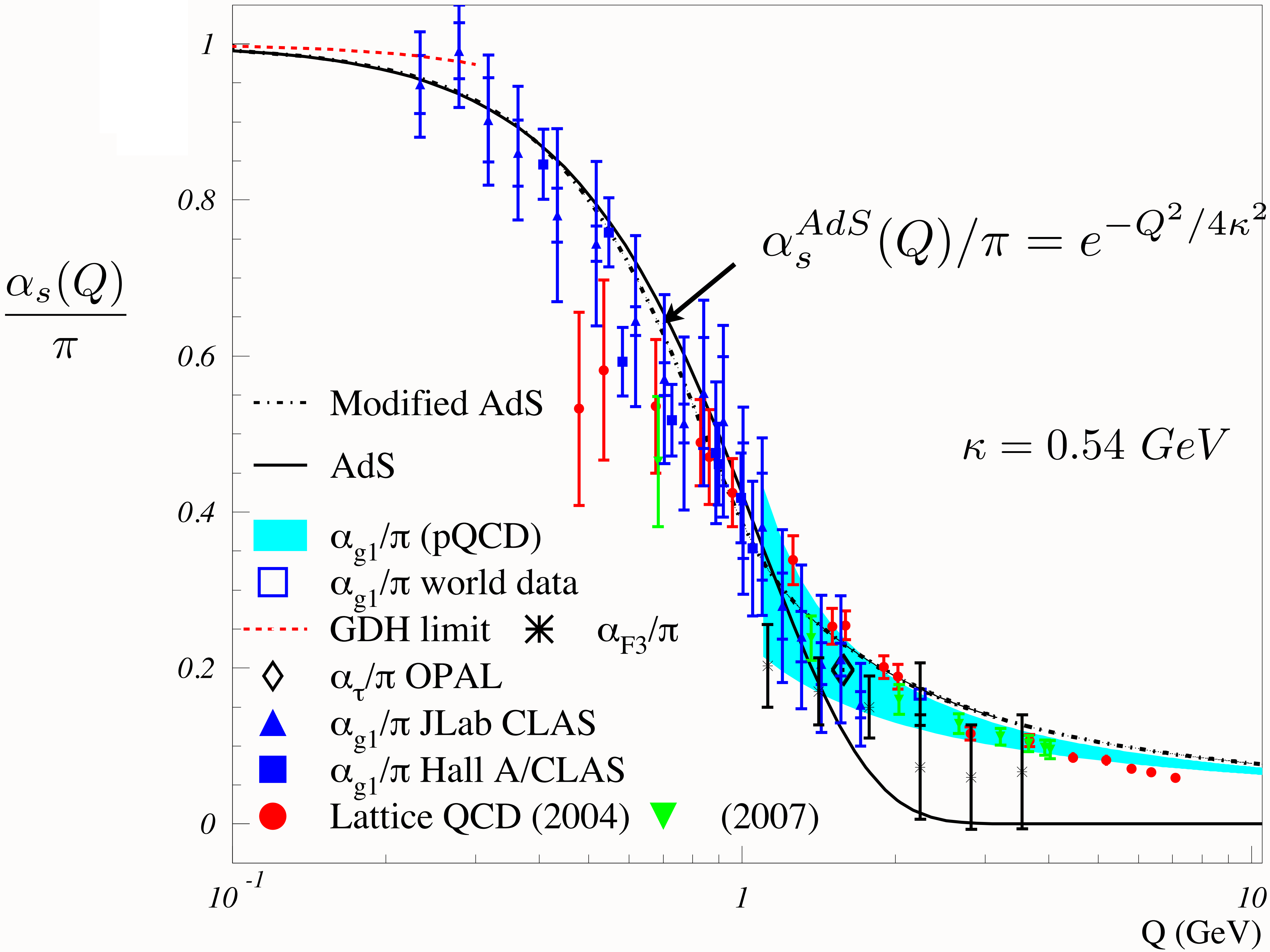
JLab data from EG1(2008), CLAS, and Hall A

α_s runs only modestly at small Q^2

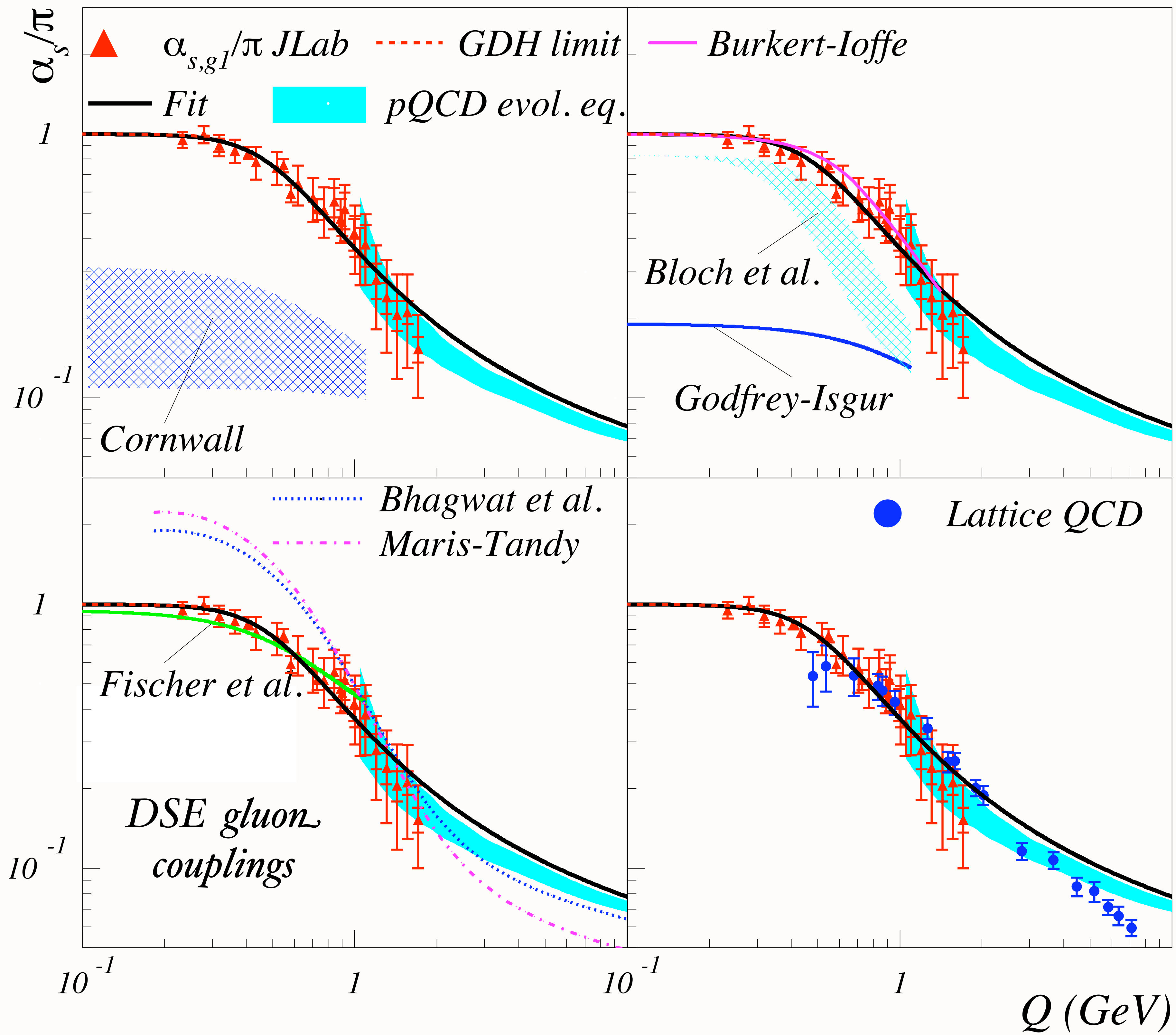
Fig. from 0803.4119, Duer et al.

Running Coupling from Light-Front Holography and AdS/QCD

Analytic, defined at all scales, IR Fixed Point



Deur, de Teramond, sjb



α_s/π
$\alpha_{s,g1}/\pi$ JLab
GDH limit
Burkert-Ioffe
Fit
pQCD evol. eq.
Cornwall
Bloch et al.
Godfrey-Isgur
Bhagwat et al.
Maris-Tandy
Fischer et al.
DSE gluon couplings
Lattice QCD
1
10^{-1}
Q (GeV)

Sublimated Gluons

- AdS/QCD soft-wall model has confining potential . Gluon exchange absent.

- Coupling falls exponentially -- misses asymptotic freedom at large Q^2

- Interpretation: Gluons sublimated into potential below 1 GeV^2 virtuality

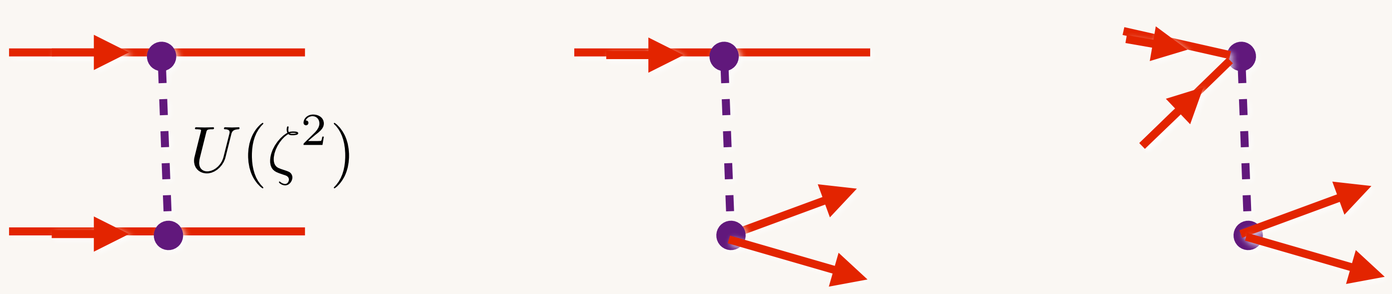
- Higher Fock states with extra quark-antiquark pairs, no gluons

Higher Fock States

- Exposed by timelike form factor through dressed current.
- Created by confining interaction

$$P^-_{\text{confinement}} \simeq \kappa^4 \int dx^- d^2\vec{x}_\perp \frac{\overline{\psi}\gamma^+ T^a \psi}{P^+} \frac{1}{(\partial/\partial_\perp)^4} \frac{\overline{\psi}\gamma^+ T^a \psi}{P^+}$$

- Similar to QCD(1+1) in lcg



Meson Transition Form-Factors

[S. J. Brodsky, Fu-Guang Cao and GdT, arXiv:1005.39XX]

- Pion TFF from 5-dim Chern-Simons structure [Hill and Zachos (2005), Grigoryan and Radyushkin (2008)]

$$\int d^4x \int dz\, \epsilon^{LMNPQ} A_L \partial_M A_N \partial_P A_Q$$

$$\sim (2\pi)^4 \delta^{(4)} \left(p_\pi + q - k\right) F_{\pi\gamma}(q^2) \epsilon^{\mu\nu\rho\sigma} \epsilon_\mu(q)(p_\pi)_\nu \epsilon_\rho(k) q_\sigma$$

- Take $A_z \propto \Phi_\pi(z)/z, \quad \Phi_\pi(z) = \sqrt{2P_{q\bar{q}}}\, \kappa\, z^2 e^{-\kappa^2 z^2/2}, \quad \langle \Phi_\pi | \Phi_\pi \rangle = P_{q\bar{q}}$
- Find $\left(\phi(x) = \sqrt{3} f_\pi x(1-x), \quad f_\pi = \sqrt{P_{q\bar{q}}}\, \kappa/\sqrt{2}\pi\right)$

$$Q^2 F_{\pi\gamma}(Q^2) = \frac{4}{\sqrt{3}} \int_0^1 dx \frac{\phi(x)}{1-x} \left[1 - e^{-P_{q\bar{q}} Q^2 (1-x)/4\pi^2 f_\pi^2\, x}\right]$$

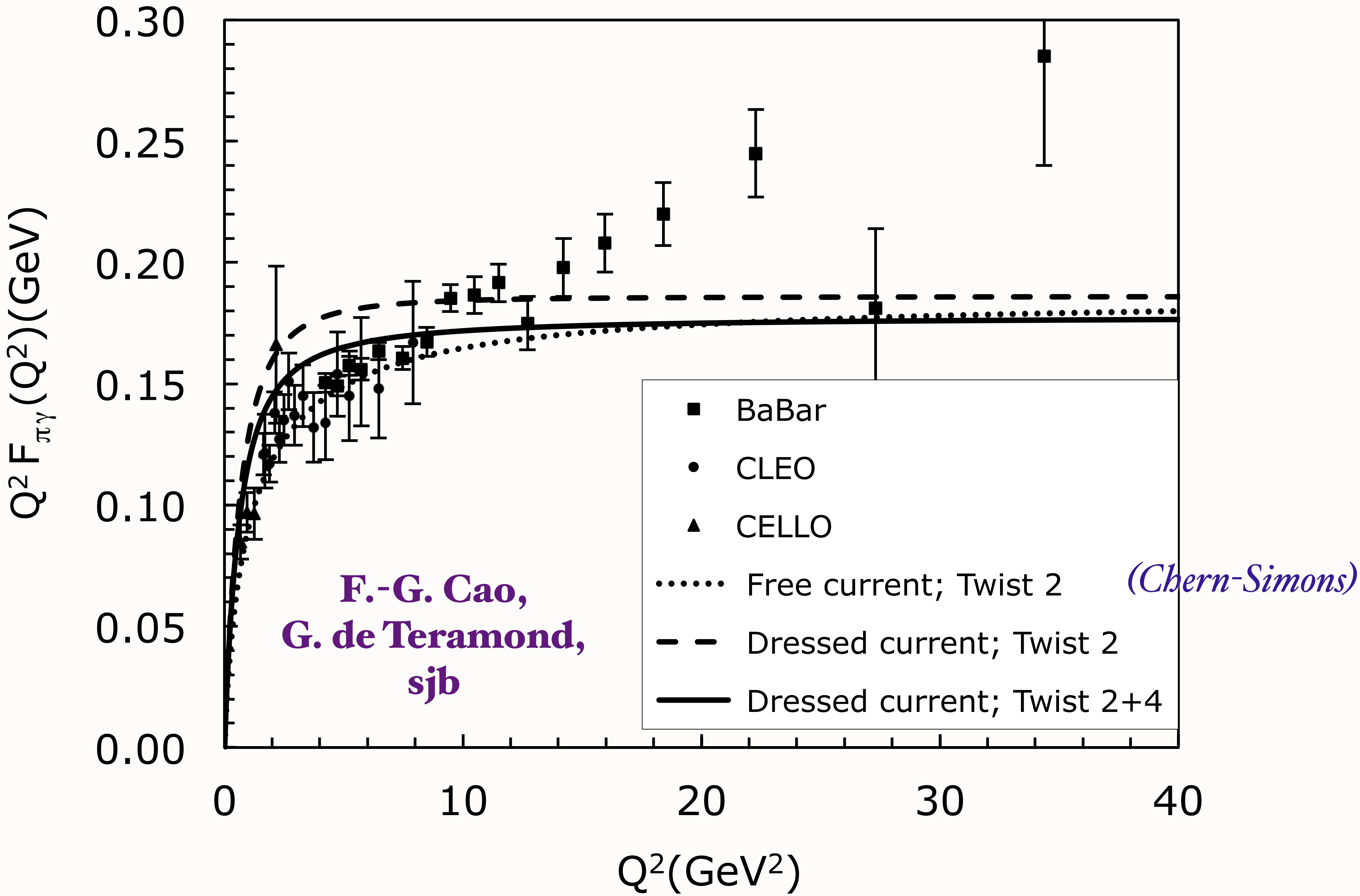
normalized to the asymptotic DA $[P_{q\bar{q}} = 1 \to$ Musatov and Radyushkin (1997)]

G.P. Lepage, sjb

- Large Q^2 TFF is identical to first principles asymptotic QCD result $Q^2 F_{\pi\gamma}(Q^2 \to \infty) = 2 f_\pi$
- The CS form is local in AdS space and projects out only the asymptotic form of the pion DA

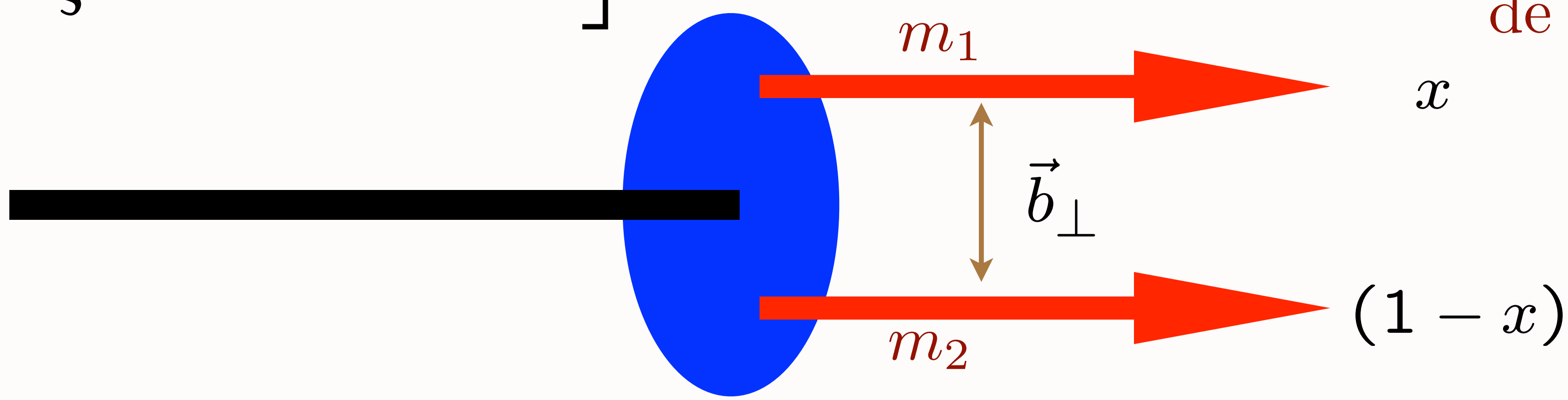
Photon-to-pion transition form factor

$$Q^2 F_{\pi\gamma}(Q^2 \to \infty) = 2 f_\pi$$



$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

de Teramond, sjb



$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$ *Holographic Variable*

$$-\frac{d}{d\zeta^2} \equiv \frac{k_\perp^2}{x(1-x)}$$ *LF Kinetic Energy in momentum space*

Assume LFWF is a dynamical function of the quark-antiquark invariant mass squared

$$-\frac{d}{d\zeta^2} \to -\frac{d}{d\zeta^2} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \equiv \frac{k_\perp^2 + m_1^2}{x} + \frac{k_\perp^2 + m_2^2}{1-x}$$

Result: Soft-Wall LFWF for massive constituents

$$\psi(x, \mathbf{k}_\perp) = \frac{4\pi c}{\kappa\sqrt{x(1-x)}} e^{-\frac{1}{2\kappa^2}\left(\frac{\mathbf{k}_\perp^2}{x(1-x)} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right)}$$

LF WF in impact space: soft-wall model with massive quarks

$$\psi(x, \mathbf{b}_\perp) = \frac{c\,\kappa}{\sqrt{\pi}} \sqrt{x(1-x)}\, e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2 - \frac{1}{2\kappa^2}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]}$$

$$z \to \zeta \to \chi$$

$$\chi^2 = b^2 x(1-x) + \frac{1}{\kappa^4}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]$$

J/ψ

LFWF peaks at

$$x_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

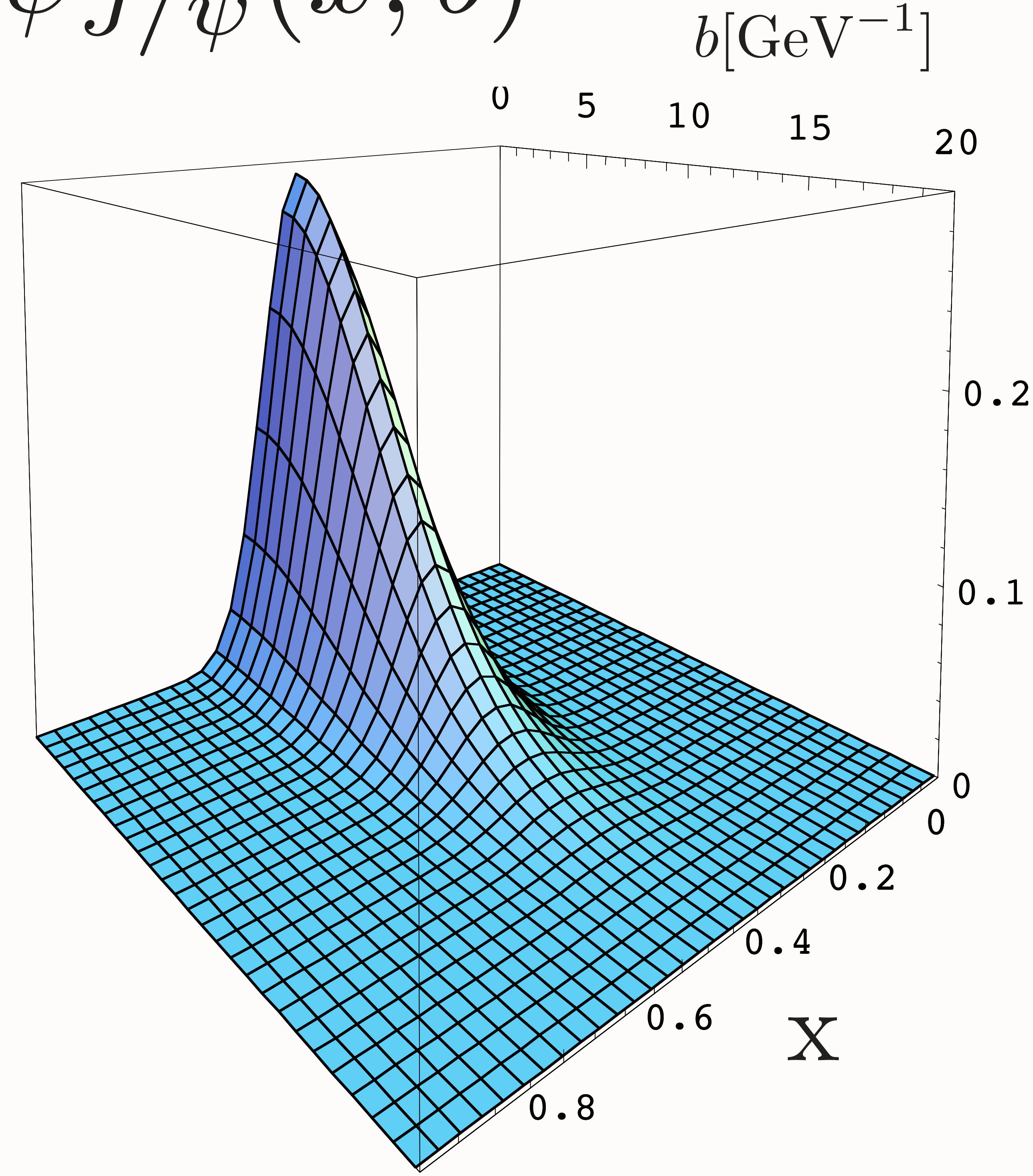
where

$$m_{\perp i} = \sqrt{m^2 + k_\perp^2}$$

minimum of LF energy denominator

$$\kappa = 0.375 \text{ GeV}$$

$$\psi_{J/\psi}(x,b)$$



$$m_a = m_b = 1.25 \text{ GeV}$$

Light and heavy mesons in a soft-wall holographic model

Valery E. Lyubovitskij[*1†], **Tanja Branz**[1], **Thomas Gutsche**[1], **Ivan Schmidt**[2], **Alfredo Vega**[2]

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Kepler Center for Astro and Particle Physics,
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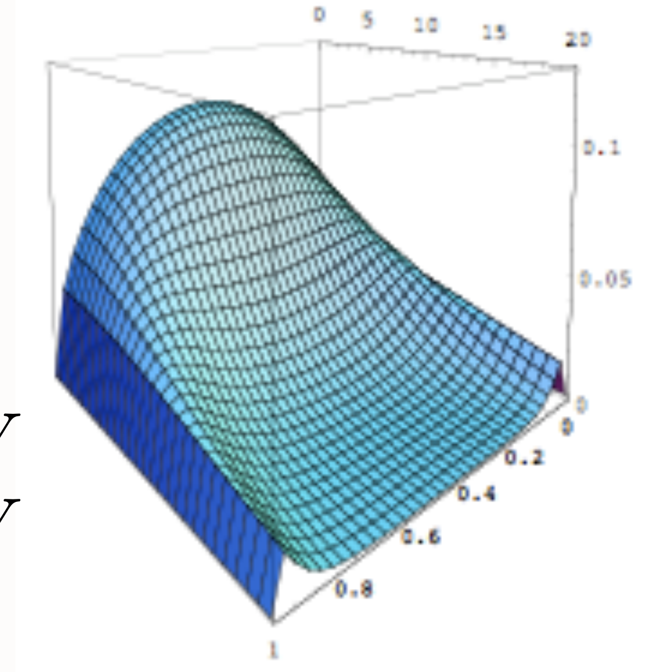
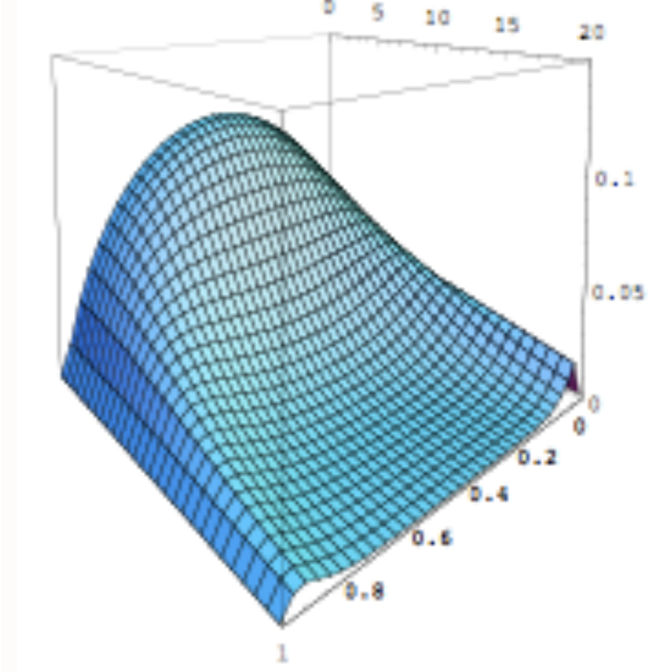
Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile

We study the spectrum and decay constants of light and heavy mesons in a soft-wall holographic approach, using the correspondence of string theory in Anti-de Sitter space and conformal field theory in physical space-time.

$$|\pi^+ >= |u\bar{d} >$$

$m_u = 2\ MeV$
$m_d = 5\ MeV$



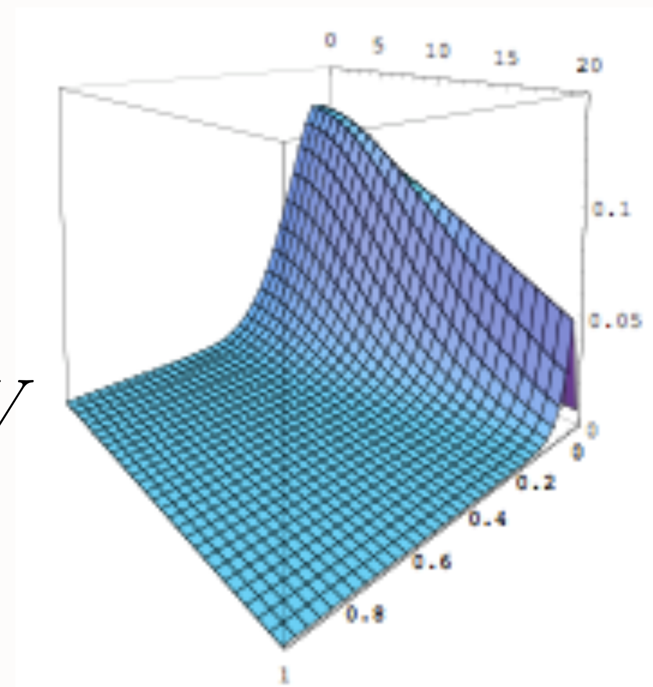


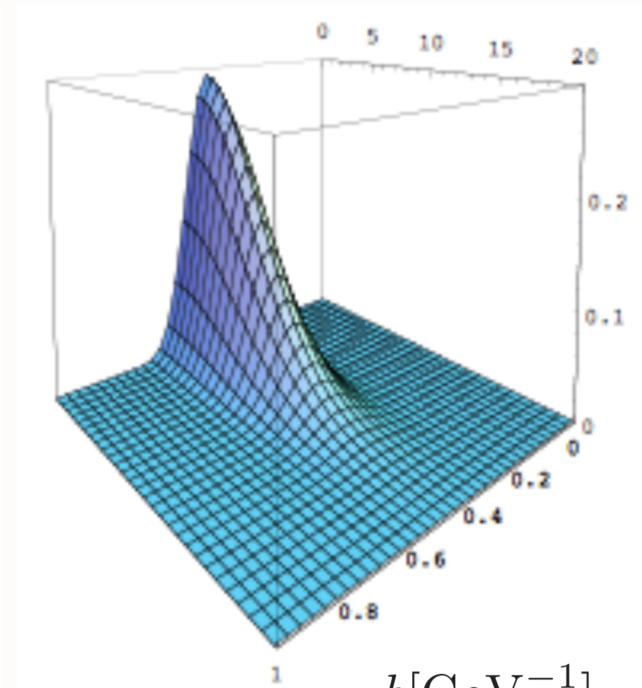
$$|K^+ >= |u\bar{s} >$$

$m_s = 95\ MeV$

$$|D^+ >= |c\bar{d} >$$

$m_c = 1.25\ GeV$

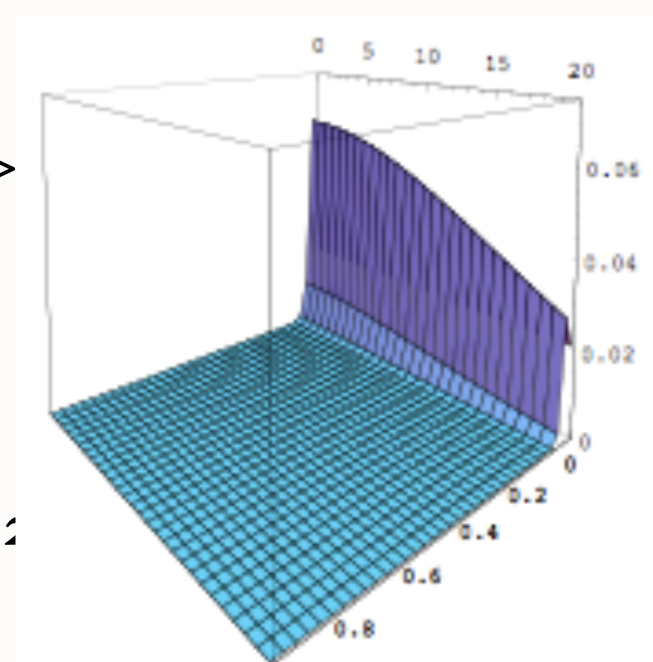


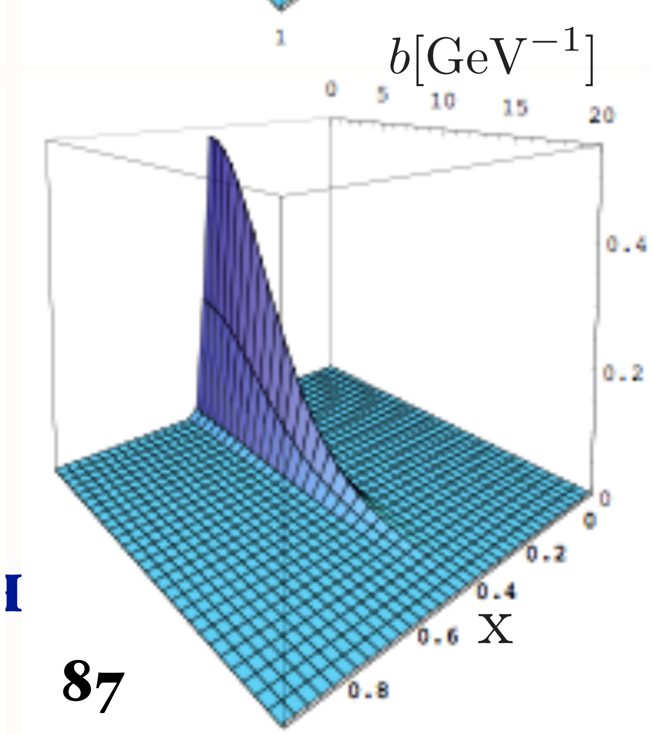


$$|\eta_c >= |c\bar{c} >$$

$$|B^+ >= |u\bar{b} >$$

$m_b = 4.2\ GeV$





$$|\eta_b >= |b\bar{b} >$$

$$\kappa = 375\ MeV$$

String Theory

↓

Mapping of Poincare' and Conformal SO (4,2) symmetries of 3+1 space to AdS5 space

AdS/CFT

↓

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering

Regge Trajectories

Conformal behavior at short distances + Confinement at large distance

AdS/QCD

QCD at the Amplitude Level

↓

Semi-Classical QCD / Wave Equations

Holography

↓

Boost Invariant 3+1 Light-Front Wave Equations

J =0,1,1/2,3/2 plus L

Integrable!

↓

Hadron Spectra, Wavefunctions, Dynamics

Future Directions

Vary
Honkanen, sjb
et al.

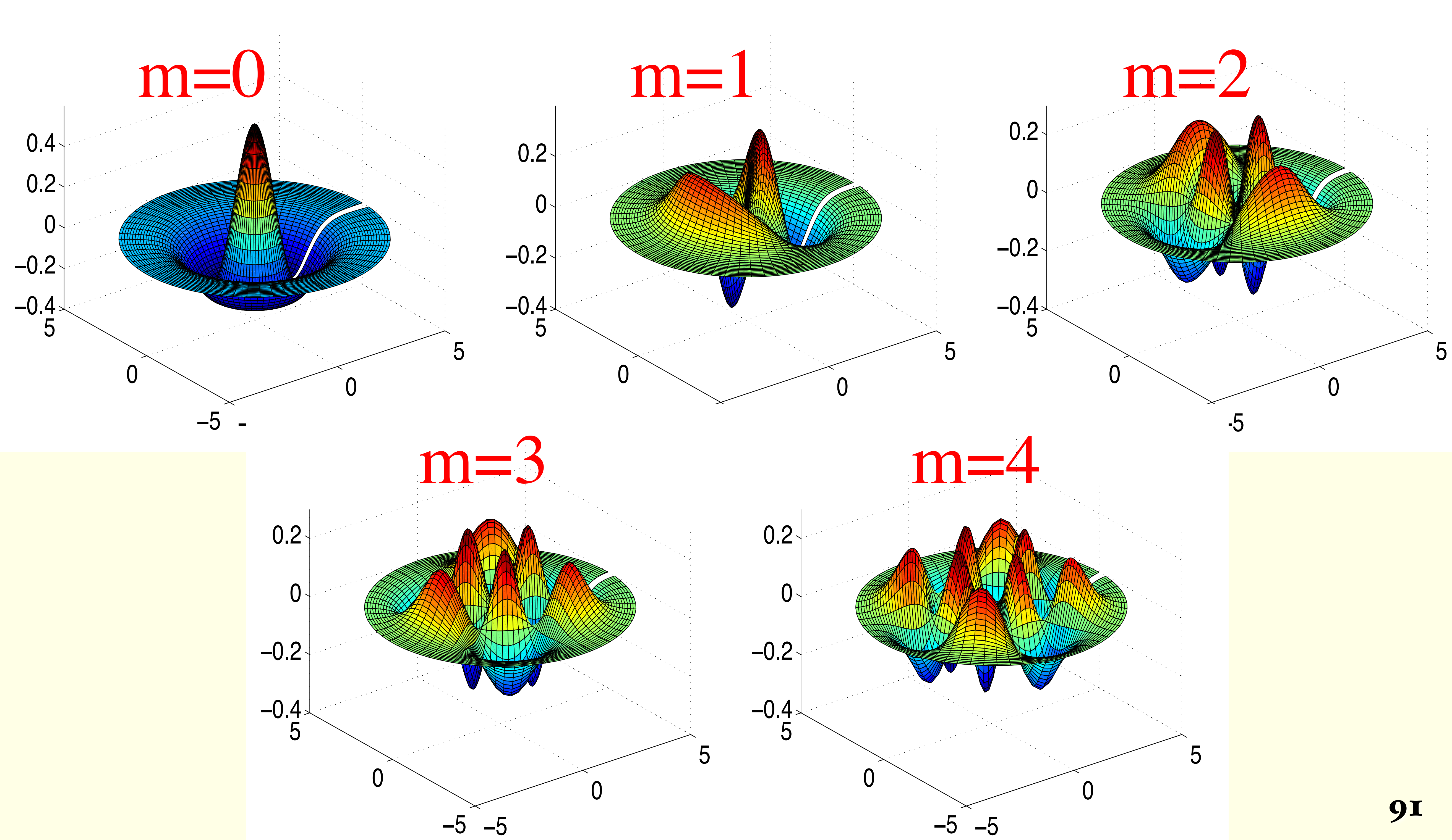
- BLFQ -- use AdS/QCD basis to diagonalize H_{LF}
- Lippmann-Schwinger -- perturbatively generate higher Fock States and systematically approach QCD *Hiller and Chabysheva*
- Hadronization at the Amplitude Level -- Off-Shell T-matrix convoluted with AdS/QCD LFWFs
- Hidden Color *C. Ji , Lepage, sjb*
- Intrinsic Heavy Quarks from confinement interaction
- Direct Processes at the LHC
- Dynamic vs. Static Structure Functions
- AdS/QCD for DVCS, Hadrons with Heavy Quarks
- LF Vacuum, In-Hadron Condensates, and the Cosmological Constant

Use AdS/CFT orthonormal Light Front Wavefunctions as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximation
- Better than plane wave basis
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations
- **Hamiltonian light-front field theory within an AdS/QCD basis.**
  J.P. Vary, H. Honkanen, Jun Li, P. Maris, A. Harindranath,
  G.F. de Teramond, P. Sternberg, E.G. Ng, C. Yang, sjb

Set of transverse 2D HO modes for $n = 1$

J.P. Vary, H. Honkanen, Jun Li, P. Maris, S.J. Brodsky, A. Harindranath, G.F. de Teramond, P. Sternberg, E.G. Ng, C. Yang, PRC



Features of Soft-Wall AdS/QCD

- **Single-variable frame-independent radial Schrodinger equation**
- **Massless pion (m_q =0)**
- **Regge Trajectories: universal slope in n and L**
- **Valid for all integer J & S.**
- **Dimensional Counting Rules for Hard Exclusive Processes**
- **Phenomenology: Space-like and Time-like Form Factors**
- **LF Holography: LFWFs; broad distribution amplitude**
- **Large Nc limit not required**
- **Add quark masses to LF kinetic energy**
- **Systematically improvable -- diagonalize H_{LF} on AdS basis**

Features of AdS/QCD LF Holography

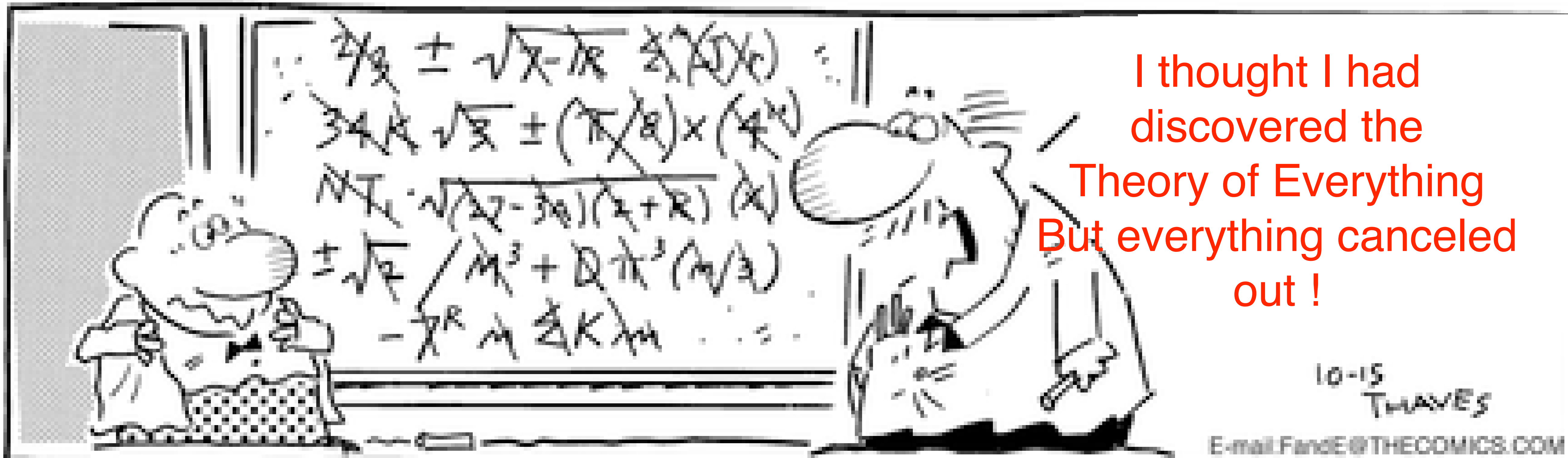
- Based on Conformal Scaling of Infrared QCD Fixed Point
- Conformal template: Use isometries of AdS5
- Interpolating operator of hadrons based on twist, superfield dimensions
- Finite Nc = 3: Baryons built on 3 quarks -- Large Nc limit not required
- Break Conformal symmetry with dilaton
- Dilaton introduces confinement -- positive exponent
- Origin of Linear and HO potentials: Stochastic arguments (Glazek); General 'classical' potential for Dirac Equation (Hoyer)
- Effective Charge from AdS/QCD at all scales
- Conformal Dimensional Counting Rules for Hard Exclusive Processes

SCIENCE VOL 265 15 SEPTEMBER 1995

A Theory of Everything Takes Place

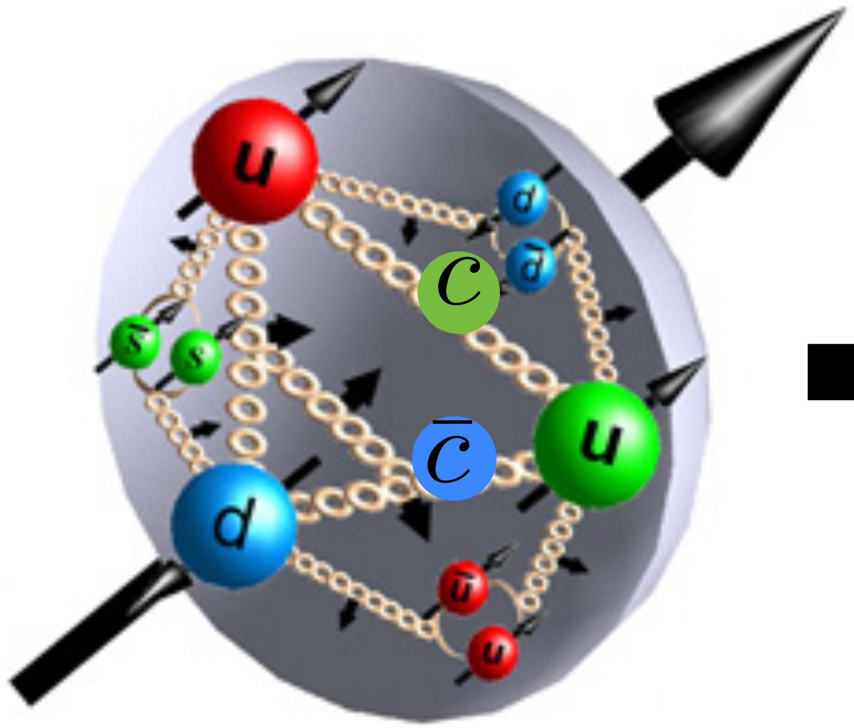
String theorists have broken an impasse and may be on their way to converting this mathematical structure -- physicists' best hope for unifying gravity and quantum theory -- into a single coherent theory.

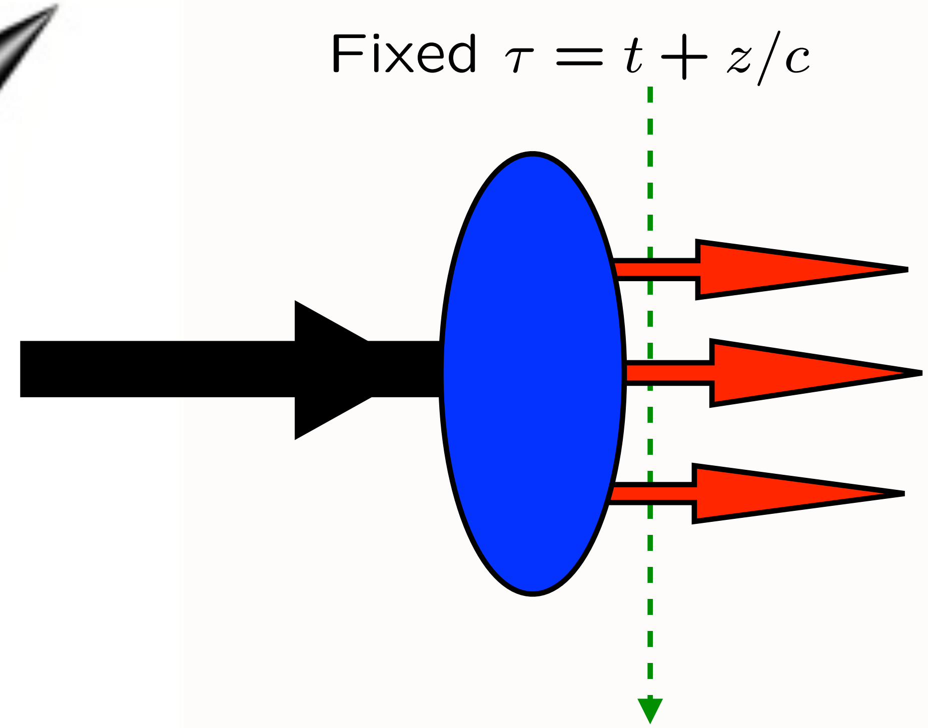
Frank and Ernest

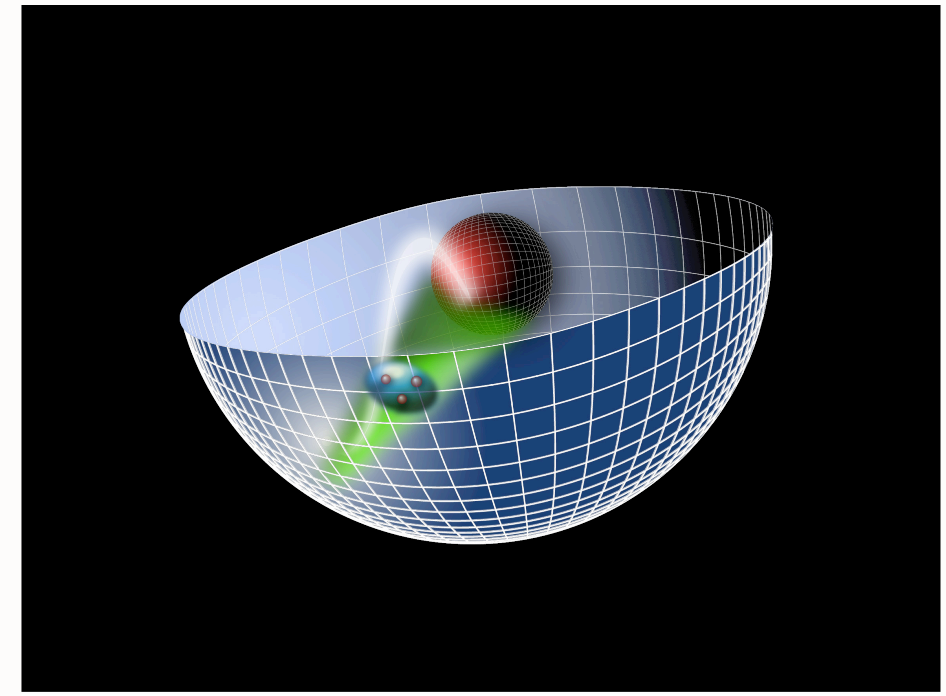


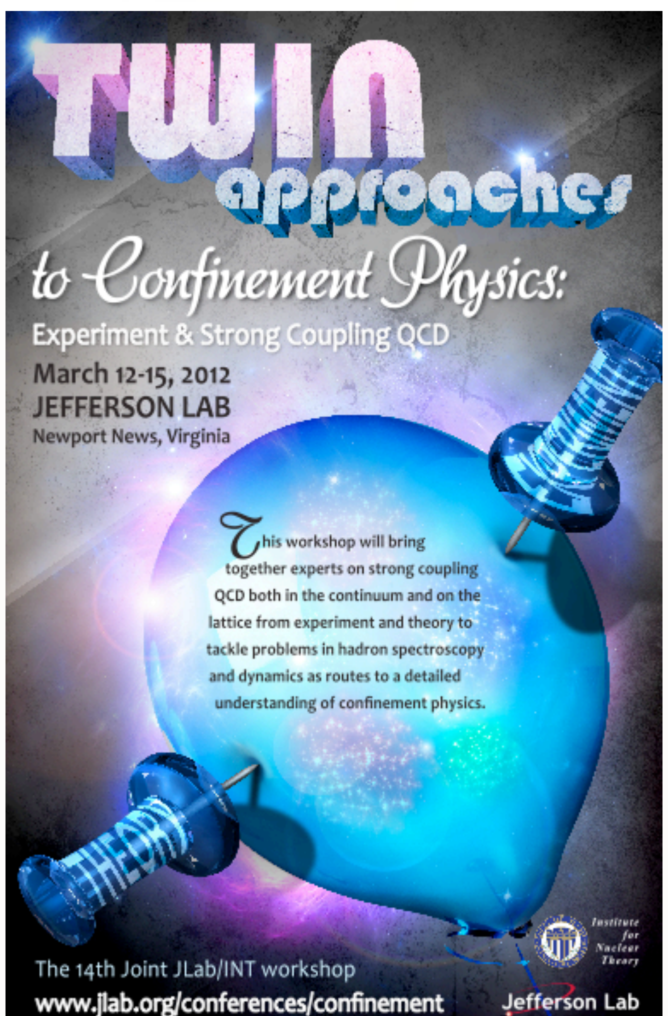
AdS/QCD and Light-Front Holography:

A Novel Approach to Confinement and Non-Perturbative QCD









Workshop on Confinement Physics

March 12-15, 2012

Thomas Jefferson National Accelerator Facility

Newport News, VA

Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY